論　　説

Some Properties of Focus Points in One-Shot Decision Theory

Peijun Guo

Abstract- One-shot (one-time) decision problems concern the situations where a decision is experienced only once. Such one-shot decision problems are commonly encountered in business, economics and social systems. One-shot decision theory has been initially proposed by Guo [4]. The one-shot decision procedure comprises two steps. In the first step, a decision maker identifies which state of nature should be taken into account for each alternative amongst all available states of nature. These identified states of nature are called focus points. In the second step, a decision maker evaluates the alternatives based on focus points where the consequences provided by all alternatives are compared with each other to obtain the best alternative. In this paper, the properties of focus points are given and the detailed proofs are shown.

Keywords - Focus points, satisfaction function, one-shot decision, possibility distribution

1 Introduction

One-shot (one-time) decision is typical for situations where a decision is made only once. This type of a decision making problem has been receiving increasing research interest because of the growing dominance of service industries for which one-shot decision problem is especially applicable. Other typical examples of one-shot decisions include mergers and acquisitions (M&A), new product development, supply chain management of products with short life cycles, and emergency management for irregular events.

In [4], the one-shot decision theory is initially proposed. The one-shot decision procedure comprises two steps. In the first step, a decision maker identifies which state of nature should be taken into account for each alternative amongst all available states of nature. These identified states of nature are called focus points. Twelve types of focus points are proposed according to a decision maker's attitudes about possibility of a state of nature and satisfaction resulted by its occurrence. In the second step, a decision maker evaluates the alternatives based on focus points where the consequences provided by all alternatives are compared with each other to obtain the best alternative. The relationships between different focus points are analyzed. In [1], private real estate investment problem is analyzed within one-shot decision framework. The analysis demonstrates the relation between the amount of uncertainty and the investment scale for different types of personal investors. The proposed model provides insights into personal real estate investment decisions and important policy implications to regulate urban land development. In [2], a duopoly market of a new product with a short life cycle is analyzed where three kinds of firms, i.e. normal, active, and passive ones are considered. Possibilistic Cournot equilibriums are obtained for different kinds of pairs of firms in a duopoly market. The results of analysis are quite in agreement with the situations encountered in the real business world. In [3], the duopoly market with asymmetric possibilistic information is analyzed. In this paper, the properties of focus points in the one-shot decision theory are given and the proofs are shown.

This paper is organized as follows: In Section 2, the basic concepts of focus points in the one-shot decision theory are introduced. In Section 3, the properties of focus points are given and the proofs are shown in detail.

2　Focus points in the one-shot decision theory

In one-shot decision problems, the set of an alternative a is A and the set of a state of nature x is S. The degree to which a state of nature is to occur in the future is characterized by a possibility distribution $\pi(x)$ defined below.

Definition 1. Given a function $\pi: S \to [0,1]$ if $\max_{x\in S} \pi(x)=1$, then $\pi(x)$ is called a possibility distribution where S is the sample space. $\pi(x)$ is the possibility degree of x.

$\pi(x)=1$ means that it is normal that x occurs and $\pi(x)=0$ means that it is abnormal that x occurs. The smaller the possibility degree of x, the more surprising is the occurrence of x.

The consequence resulting from the combination of an alternative a and a state of nature x is referred to as a payoff, denoted as $v(x,a)$. The satisfaction level of a decision maker for a payoff can be expressed by a satisfaction function, as defined below.

Definition 2. Denote the set of a payoff $v(x,a)$ as V. The function $u: V \to [0,1]$ is called a satisfaction function if it satisfies $u(v_1)>u(v_2)$ for $v_1>v_2$.

Because the payoff is the function of x and a, we write the satisfaction function as $u(x,a)$ in this paper. For identifying the focus points, the following operators are needed.

Definition 3. Given a vector $[b_1, b_2, \cdots, b_n]$, $\min[b_1, b_2, \cdots, b_n]$ and $\max[b_1, b_2, \cdots, b_n]$ are defined as follows:

$$\min[b_1, b_2, \cdots, b_n] = \left[\bigwedge_{i=1,\cdots,n} b_i, \bigwedge_{i=1,\cdots,n} b_i, \cdots, \bigwedge_{i=1,\cdots,n} b_i\right], \tag{1}$$

$$\max[b_1, b_2, \cdots, b_n] = \left[\bigvee_{i=1,\cdots,n} b_i, \bigvee_{i=1,\cdots,n} b_i, \cdots, \bigvee_{i=1,\cdots,n} b_i\right]. \tag{2}$$

$\min[b_1, b_2, \cdots, b_n]$ and $\max[b_1, b_2, \cdots, b_n]$ are lower and upper bounds of $[b_1, b_2, \cdots, b_n]$, respectively.

In one-shot decision theory, there are twelve types of focus points as follows [4]:

Type I : $x_\alpha^{1*}(a) = \arg\max_{x\in X^{\geq\alpha}} u(x,a)$ where $X^{\geq\alpha} = \{x \mid \pi(x) \geq \alpha\}$.

The given parameter α is a level used to distinguish whether the possibility degree is evaluated as 'high' by a decision maker. If $\alpha = 1$ then only the normal case $(\pi(x) = 1)$ is considered. The states of nature belonging to $X^{\geq\alpha} = \{x \mid \pi(x) \geq \alpha\}$ are regarded as having the equivalent possibility to occur. $x_\alpha^{1*}(a)$ is the most favorable (the satisfaction level is the highest) state of nature that has a high possibility to occur. $x_\alpha^{1*}(a)$ is Type I focus point.

Type II : $x_\alpha^{2*}(a) = \arg \min_{x \in X^{\geq\alpha}} u(x, a)$ where $X^{\geq\alpha} = \{x \mid \pi(x) \geq \alpha\}$,

which called as Type II focus point, is the most unpleasant (the satisfaction level is the lowest) state of nature that has a high possibility to occur.

Type III : $x_\alpha^{3*}(a) = \arg \max_{x \in X^{\leq\alpha}} u(x, a)$ where $X^{\leq\alpha} = \{x \mid \pi(x) \leq \alpha\}$,

which called as Type III focus point, is the most favorable state of nature that has a low possibility to occur.

Type IV : $x_\alpha^{4*}(a) = \arg \min_{x \in X^{\leq\alpha}} u(x, a)$ where $X^{\leq\alpha} = \{x \mid \pi(x) \leq \alpha\}$,

which called as Type IV focus point, is the most unpleasant state of nature that has a low possibility to occur.

Type V : $x_\beta^{5*}(a) = \arg \max_{x \in X^{\geq\beta}(a)} \pi(x)$ where $X^{\geq\beta}(a) = \{x \mid u(x, a) \geq \beta\}$.

The given parameter β is the level to distinguish whether the satisfaction level is evaluated as 'high' by a decision maker. When $\beta = 0$, $x_\beta^{5*}(a)$ is the state of nature whose possibility degree is 1. The states of nature belonging to $X^{\geq\beta}(a) = \{x \mid u(x, a) \geq \beta\}$ are regarded as having the equivalent satisfaction levels provided by an alternative a. $x_\beta^{5*}(a)$ is the favorable (the satisfaction level is high) state of nature that has the highest possibility to occur. $x_\beta^{5*}(a)$ is Type V focus point.

Type VI : $x_\beta^{6*}(a) = \arg \min_{x \in X^{\geq\beta}(a)} \pi(x)$ where $X^{\geq\beta}(a) = \{x \mid u(x, a) \geq \beta\}$,

which called as Type VI focus point, is a favorable state of nature that has the smallest possibility to occur.

Type VII : $x_\beta^{7*}(a) = \arg \max_{x \in X^{\leq\beta}(a)} \pi(x)$ where $X^{\leq\beta}(a) = \{x \mid u(x, a) \leq \beta\}$,

which called as Type VII focus point, is the unpleasant (the satisfaction level is low) state of nature that has the highest possibility to occur.

Type VIII : $x_\beta^{8*}(a) = \arg \min_{x \in X^{\leq\beta}(a)} \pi(x)$ where $X^{\leq\beta}(a) = \{x \mid u(x, a) \leq \beta\}$,

which called as Type VIII focus point, is the unpleasant state of nature that has the smallest possibility to occur.

Type IX : $x^{9*}(a) = \arg\max_{x\in S} \min[\pi(x), u(x,a)]$. (3)

$\arg\max_{x\in S} \min[\pi(x), u(x,a)]$ is for seeking the state of nature that has the higher possibility degree and the higher satisfaction level. The obtained state of nature $x^{9*}(a)$ is Type IX focus point.

Type X : $x^{10*}(a) = \arg\min_{x\in S} \max[\pi(x), u(x,a)]$. (4)

$\arg\min_{x\in S} \max[\pi(x), u(x,a)]$ is for seeking the state of nature that has the lower possibility degree and the lower satisfaction level. $x^{10*}(a)$ is Type X focus point.

Type XI : $x^{11*}(a) = \arg\min_{x\in S} \max[1-\pi(x), u(x,a)]$. (5)

$x^{11*}(a)$ is the state of nature that has the higher possibility degree and the lower satisfaction level. The obtained state of nature $x^{11*}(a)$ is Type XI focus point.

Type XII : $x^{12*}(a) = \arg\min_{x\in S} \max[\pi(x), 1-u(x,a)]$. (6)

$x^{12*}(a)$ is the state of nature that has the lower possibility degree and the higher satisfaction level. $x^{12*}(a)$ is Type XII focus point.

For one alternative, more than one state of nature might exist as one type of focus point. We denote the sets of twelve types of focus points of the alternative a as $X_\alpha^1(a)$, $X_\alpha^2(a)$, $X_\alpha^3(a)$, $X_\alpha^4(a)$, $X_\beta^5(a)$, $X_\beta^6(a)$, $X_\beta^7(a)$, $X_\beta^8(a)$, $X^9(a)$, $X^{10}(a)$, $X^{11}(a)$, and $X^{12}(a)$, respectively.

3 The Properties of Focus Points

Let us give some properties of focus points.

Theorem 1.

$$\arg\max_{x\in S} \min(1-\pi(x), u(x,a)) = \arg\min_{x\in S} \max(\pi(x), 1-u(x,a)) \tag{7}$$

$$\arg\max_{x\in S} \min(\pi(x), 1-u(x,a)) = \arg\min_{x\in S} \max(1-\pi(x), u(x,a)) \tag{8}$$

Proof. Without loss of generality, we assume $\max_x \min(1-\pi(x), u(x,a)) = 1-\pi(x_e)$ which leads to

$$1-\pi(x_e) \geq \min(1-\pi(x), u(x,a)), \tag{9}$$

and

$$1-\pi(x_e) \leq u(x_e, a). \tag{10}$$

Considering (9), we have

$$\pi(x_e) \leq 1-\min(1-\pi(x), u(x,a)),$$

which can be rewritten as

$$\pi(x_e) \leq \max(\pi(x), 1-u(x,a)). \tag{11}$$

(10) can be rewritten as

$$\pi(x_e) \geq 1-u(x_e, a). \tag{12}$$

(11) and (12) implies

$$\min_{x \in S} \max(\pi(x), 1-u(x,a)) = \pi(x_e),$$

so that (7) is correct. Similarly, we can prove (8). □

Theorem 2.

(I) $X_\alpha^1(a) \cup X_\beta^5(a) \subseteq X^9(a)$, (13)

where

$$\alpha = \beta = \max_{x \in S} \min(\pi(x), u(x,a)). \tag{14}$$

(II) $X_\alpha^4(a) \cup X_\beta^8(a) \subseteq X^{10}(a)$, (15)

where

$$\alpha = \beta = \min_{x \in S} \max(\pi(x), u(x,a)). \tag{16}$$

(III) $X_\alpha^2(a) \cup X_\beta^7(a) \subseteq X^{11}(a)$, (17)

where

$$\alpha = 1-\beta = \max_{x \in S} \min(\pi(x), 1-u(x,a)). \tag{18}$$

(IV) $X_\alpha^3(a) \cup X_\beta^6(a) \subseteq X^{12}(a)$, (19)

where

$$1-\alpha = \beta = \max_{x \in S} \min(1-\pi(x), u(x,a)). \tag{20}$$

Proof.

(I) Assume that

$$x^o \in X_\alpha^1(a), \tag{21}$$

where α satisfies (14). With considering x^{9*}, it is clear that $\exists x \in X^{\geq \alpha}$ such that

$$u(x,a) \geq \max_{x \in S} \min(\pi(x), u(x,a)).$$

(21) implies that

$$\pi(x^o) \geq \max_{x \in S} \min(\pi(x), u(x,a)),$$

$$u(x^o, a) \geq \max_{x \in S} \min(\pi(x), u(x, a)),$$

which means

$$\min(\pi(x^o), u(x^o, a)) \geq \max_{x \in S} \min(\pi(x), u(x, a)).$$

As

$$\min(\pi(x^o), u(x^o, a)) \leq \max_{x \in S} \min(\pi(x), u(x, a)),$$

we have

$$\min(\pi(x^o), u(x^o, a)) = \max_{x \in S} \min(\pi(x), u(x, a)),$$

which means that

$$x^o \in X^9(a). \tag{22}$$

(21) and (22) lead to $X_\alpha^1(a) \subseteq X^9(a)$. Likewise, we can prove $X_\beta^5(a) \subseteq X^9(a)$. Thus, (13) holds.

(II) Assume that

$$x^o \in X_\alpha^{4*}(a), \tag{23}$$

where α satisfies (16). With considering x^{10*}, it is clear that $\exists x \in X^{\leq \alpha}$ such that

$$u(x, a) \leq \min_{x \in S} \max(\pi(x), u(x, a)). \tag{24}$$

From (23), we know

$$\pi(x^o) \leq \min_{x \in S} \max(\pi(x), u(x, a)), \tag{25}$$

$$u(x^o, a) \leq \min_{x \in S} \max(\pi(x), u(x, a)), \tag{26}$$

which means

$$\max(\pi(x^o), u(x^o, a)) \leq \min_{x \in S} \max(\pi(x), u(x, a)). \tag{27}$$

As

$$\max(\pi(x^o), u(x^o, a)) \geq \min_{x \in S} \max(\pi(x), u(x, a)), \tag{28}$$

we have

$$\max(\pi(x^o), u(x^o, a)) = \min_{x \in S} \max(\pi(x), u(x, a)), \tag{29}$$

which means

$$x^{o} \in X^{10*}(a). \tag{30}$$

(23) and (30) lead to $X_{\alpha}^{4}(a) \subseteq X^{10*}(a)$. Likewise, we can prove $X_{\beta}^{8*}(a) \subseteq X^{10*}(a)$. Thus, (15) holds.

(III) Assume that

$$x^{o} \in X_{\alpha}^{2*}(a), \tag{31}$$

where α satisfies (18). With considering x^{11*}, it is clear that $\exists x \in X^{\geq \alpha}$ such that

$$u(x,a) \leq \min_{x \in S} \max(1-\pi(x), u(x,a)) = 1-\max_{x \in S} \min(\pi(x), 1-u(x,a)). \tag{32}$$

From (31), we know

$$1-\pi(x^{o}) \leq 1-\max_{x \in S} \min(\pi(x), 1-u(x,a)), \tag{33}$$

$$u(x^{o}, a) \leq 1-\max_{x \in S} \min(\pi(x), 1-u(x,a)), \tag{34}$$

which means

$$\max(1-\pi(x^{o}), u(x^{o}, a)) \leq 1-\max_{x \in S} \min(\pi(x), 1-u(x,a)) = \min_{x \in S} \max(1-\pi(x), u(x,a)). \tag{35}$$

As

$$\max(1-\pi(x^{o}), u(x^{o}, a)) \geq \min_{x \in S} \max(1-\pi(x), u(x,a)), \tag{36}$$

we have

$$\max(1-\pi(x^{o}), u(x^{o}, a)) = \min_{x \in S} \max(1-\pi(x), u(x,a)), \tag{37}$$

which means that

$$x^{o} \in X^{11*}(a). \tag{38}$$

(31) and (38) lead to

$$X_{\alpha}^{2*}(a) \subseteq X^{11*}(a) \tag{39}$$

Assume that

$$x^{o} \in X_{\beta}^{7*}(a), \tag{40}$$

where β satisfies (18). With considering x^{11*}, it is clear that $\exists x \in X^{\leq \beta}$ such that

$$\pi(x) \geq \max_{x \in S} \min(\pi(x), 1-u(x,a)). \tag{41}$$

$$(1-\pi(x) \leq 1-\max_{x \in S} \min(\pi(x), 1-u(x,a)) = \min_{x \in S}(1-\pi(x), u(x,a)))$$

From (40), we know

$$\pi(x^o) \geq \max_{x \in S} \min(\pi(x), 1-u(x,a)), \tag{42}$$

$$u(x^o, a) \leq 1-\max_{x \in S} \min(\pi(x), 1-u(x,a)). \tag{43}$$

(43) can be rewritten as

$$1-u(x^o, a) \geq \max_{x \in S} \min(\pi(x), 1-u(x,a)). \tag{44}$$

(42) and (44) leads to

$$\min(\pi(x^o), 1-u(x^o, a)) \geq \max_{x \in S} \min(\pi(x), 1-u(x,a)). \tag{45}$$

As

$$\min(\pi(x^o), 1-u(x^o, a)) \leq \max_{x \in S} \min(\pi(x), 1-u(x,a)), \tag{46}$$

we have

$$\min(\pi(x^o), 1-u(x^o, a)) = \max_{x \in S} \min(\pi(x), 1-u(x,a)). \tag{47}$$

It follows from (5) and (8) that

$$x^o \in X^{11*}(a). \tag{48}$$

(40) and (48) lead to

$$X_\beta^{7*}(a) \subseteq X^{11*}(a). \tag{49}$$

(39) and (49) make (17) hold.

(IV) Assume that

$$x^o \in X_\beta^{6*}(a), \tag{50}$$

where β satisfies (20). With considering x^{12*}, it is clear that $\exists x \in X^{\geq \beta}$ such that

$$\pi(x) \leq \min_{x \in S} \max(\pi(x), 1-u(x,a)) = 1-\max_{x \in S} \min(1-\pi(x), u(x,a)). \tag{51}$$

From (50), we know

$$\pi(x^o) \leq 1-\max_{x \in S} \min(1-\pi(x), u(x,a)), \tag{52}$$

$$1-u(x^o, a) \leq 1-\max_{x \in S} \min(1-\pi(x), u(x,a)), \tag{53}$$

which means

$$\max(\pi(x^o), 1-u(x^o, a)) \leq 1-\max_{x \in S} \min(1-\pi(x), u(x,a)) = \min_{x \in S} \max(\pi(x), 1-u(x,a)). \tag{54}$$

As

$$\max\left(\pi(x^o), 1-u(x^o,a)\right) \geq \min_{x\in S} \max(\pi(x), 1-u(x,a)), \tag{55}$$

we have

$$\max\left(\pi(x^o), 1-u(x^o,a)\right) = \min_{x\in S} \max(\pi(x), 1-u(x,a)). \tag{56}$$

It follows from (6) that

$$x^o \in X^{12*}(a). \tag{57}$$

(50) and (57) lead to

$$X_\beta^{6*}(a) \subseteq X^{12*}(a). \tag{58}$$

Assume that

$$x^o \in X_\alpha^{3*}(a), \tag{59}$$

where α satisfies (20). With considering x^{12*}, it is clear that $\exists x \in X^{\leq\alpha}$ such that

$$u(x,a) \geq \max_{x\in S} \min(1-\pi(x), u(x,a)). \tag{60}$$

$$(1-u(x,a) \leq 1-\max_{x\in S} \min(1-\pi(x), u(x,a)) = \min_{x\in S} \max(\pi(x), 1-u(x,a)))$$

From (59), we know

$$\pi(x^o) \leq 1-\max_{x\in S} \min(1-\pi(x), u(x,a)), \tag{61}$$

$$u(x^o,a) \geq \max_{x\in S} \min(1-\pi(x), u(x,a)). \tag{62}$$

(61) can be rewritten as

$$1-\pi(x^o) \geq \max_{x\in S} \min(1-\pi(x), u(x,a)). \tag{63}$$

(62) and (63) lead to

$$\min\left(1-\pi(x^o), u(x^o,a)\right) \geq \max_{x\in S} \min(1-\pi(x), u(x,a)). \tag{64}$$

As

$$\min\left(1-\pi(x^o), u(x^o,a)\right) \leq \max_{x\in S} \min(1-\pi(x), u(x,a)), \tag{65}$$

we have

$$\min(1-\pi(x^o), u(x^o, a)) = \max_{x \in S} \min(1-\pi(x), u(x, a)). \tag{66}$$

$$(1-\min(1-\pi(x^o), u(x^o, a)) = 1-\max_{x \in S} \min(1-\pi(x), u(x, a)),$$

$$\max(\pi(x^o), 1-u(x^o, a)) = \min_{x \in S} \max(\pi(x), 1-u(x, a)))$$

It follows from (6) that

$$x^o \in X^{12*}(a). \tag{67}$$

(59) and (67) lead to

$$X_\alpha^{3*}(a) \subseteq X^{12*}(a). \tag{68}$$

(58) and (68) make (19) hold. □

Lemma 1. No other ordered pair $(\pi(x), u(x, a))$ strongly dominates the ordered pair $(\pi(x^{9*}(a)), u(x^{9*}(a), a))$; that is, $\pi(x) > \pi(x^{9*}(a))$ and $u(x, a) > u(x^{9*}(a), a)$ do not hold.

Lemma 2. For any ordered pair $(\pi(x), u(x, a))$, if $\pi(x) > \pi(x^{11*}(a))$ then the relation $u(x, a) \geq u(x^{11*}(a), a)$ holds.

Lemma 3. For any ordered pair $(\pi(x), u(x, a))$, if $u(x, a) > u(x^{12*}(a), a)$, then the relation $\pi(x) \geq \pi(x^{12*}(a))$ holds.

Theorem 2.

(I) For an alternative a, the following relation does not hold

$$u(x^{9*}(a), a) < u(x^{11*}(a), a). \tag{69}$$

(II) If $X^9(a) \cap X^{12}(a) = \emptyset$ then the following relation does not holds

$$\pi(x^{9*}(a)) < \pi(x^{12*}(a)). \tag{70}$$

Proof. If (69) holds, there are three cases; that is,

Case 1: $u(x^{9*}(a), a) < u(x^{11*}(a), a)$ for $\pi(x^{9*}(a)) < \pi(x^{11*}(a))$,

Case 2: $u(x^{9*}(a), a) < u(x^{11*}(a), a)$ for $\pi(x^{9*}(a)) > \pi(x^{11*}(a))$,

Case 3: $u(x^{9*}(a), a) < u(x^{11*}(a), a)$ for $\pi(x^{9*}(a)) = \pi(x^{11*}(a))$.

Lemma 1 shows that Case 1 is impossible to exist. Lemma 2 shows that Case 2 is impossible to exist. For Case 3, we can consider the following three cases:

Case A: $1-\pi(x^{9*}(a)) = 1-\pi(x^{11*}(a)) \leq u(x^{9*}(a), a) < u(x^{11*}(a), a)$,

Case B: $u(x^{9*}(a), a) < 1-\pi(x^{9*}(a)) = 1-\pi(x^{11*}(a)) < u(x^{11*}(a), a)$,

Case C: $u(x^{9*}(a), a) < u(x^{11*}(a), a) \leq 1-\pi(x^{9*}(a)) = 1-\pi(x^{11*}(a))$. (71)

(5) shows that Cases A and B are impossible. Considering Definition 1, we assume $\pi(x_c) = 1$ so that $1-\pi(x_c) = 0$. (3) leads to

$$u(x_c, a) \leq u(x^{9*}(a), a). \tag{72}$$

(71) and (72) leads to

$$u(x_c, a) < u(x^{11*}(a), a),$$

which is impossible with considering Lemma 2. So that (71) is not correct. As a result, (69) does not hold. If (70) holds, then there are the following three cases; that is,

Case 1: $\pi(x^{9*}(a))<\pi(x^{12*}(a))$ for $u(x^{9*}(a),a)<u(x^{12*}(a),a)$,

Case 2: $\pi(x^{9*}(a))<\pi(x^{12*}(a))$ for $u(x^{9*}(a),a)>u(x^{12*}(a),a)$,

Case 3: $\pi(x^{9*}(a))<\pi(x^{12*}(a))$ for $u(x^{9*}(a),a)=u(x^{12*}(a),a)$.

Lemma 1 shows that Case 1 is impossible to exist. Lemma 3 shows that Case 2 is impossible to exist. Case 3 is possible to hold under the following condition

$$u(x^{9*}(a),a)=u(x^{12*}(a),a)\leq\pi(x^{9*}(a))<\pi(x^{12*}(a))\leq 1-u(x^{9*}(a),a)=1-u(x^{12*}(a),a), \quad (73)$$

which is excluded by $X^{9}(a)\cap X^{12}(a)=\varnothing$. As a result, $\pi(x^{9*}(a))<\pi(x^{12*}(a))$ does not hold. □

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〔Peijun Guo, Associate Professor, Faculty of Business Administration,Yokohama National University〕

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