Focus Theory of Choice – A fundamental alternative for decision under uncertainty

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The Symposium on Artificial Intelligence and Management Analytics, Yokohama, Japan
June 30th, 2019

Agenda

1. **Focus theory of choice (FTC)**
2. **Axiomatization**
3. **Resolving the Ellsberg paradoxes**
4. **Psychological evidences**
5. **Concluding Remarks**

Focus theory of choice (FTC)

A decision maker (DM) is endowed with two distinct evaluation systems:

a positive evaluation system (PES);

a negative evaluation system (NES).

For the DM, one system is apparent and the other is latent. Which system is apparent is strongly related to the DM’s personality traits (e.g., optimistic, pessimistic).

It can also be strongly influenced by the framing.

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In the PES, for each lottery, the event which brings about a relatively high outcome with a relatively high probability has a relatively high accessibility. Such an event generates the individual's overall impression of this lottery. We call this the positive focus of this lottery.

Then, based on the positive foci of all lotteries, the best lottery is chosen.

Choosing an action owing the most attractive event.

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In the NES, an event which leads to a relatively low outcome with a relatively high probability has a relatively high accessibility. This event will generate the impression of the lottery. We call this the negative focus of this lottery.

Then, based on the negative foci of all lotteries, the best lottery is chosen.

Choosing an action owing the most acceptable event.

Example: Decision under ignorance

The event which makes an action generate the highest outcome is the positive focus of this action because it is the most attractive event for this action;

The DM chooses such an action that produces the highest outcome from among all positive foci.

This is exactly the maximax criterion.

The event which makes an action yield the lowest outcome is the negative focus of this action because it is the most concerned event for this action; the DM then chooses from the negative foci the one with the highest outcome.

This is just the maximin criterion.

Axiomatization

Positive Evaluation System (PES)

Axiom 1- Decidability: For each action, a DM can choose the most attractive event from its set of events.

Axiom 1 postulates that a DM is able to select the most attractive event from among all events of each action. Meanwhile, it implies that the most attractive event is not necessarily derived from a pairwise comparison where completeness and transitivity are needed.

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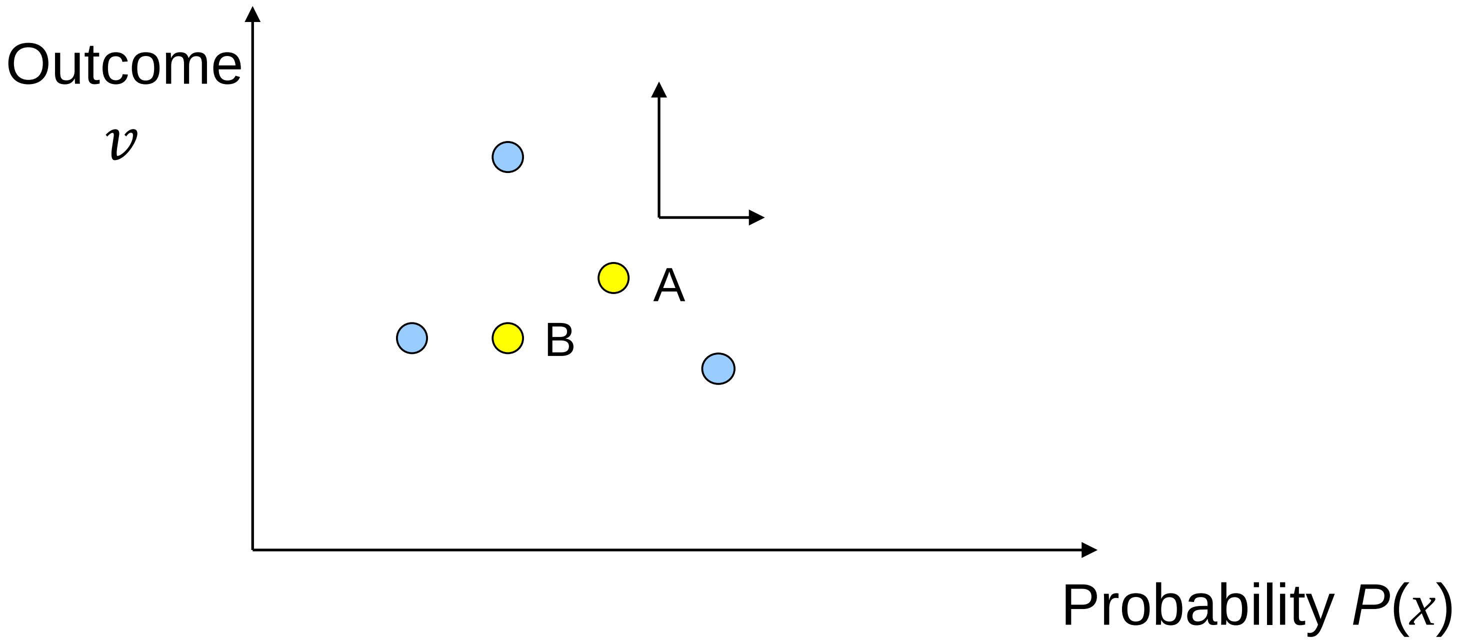
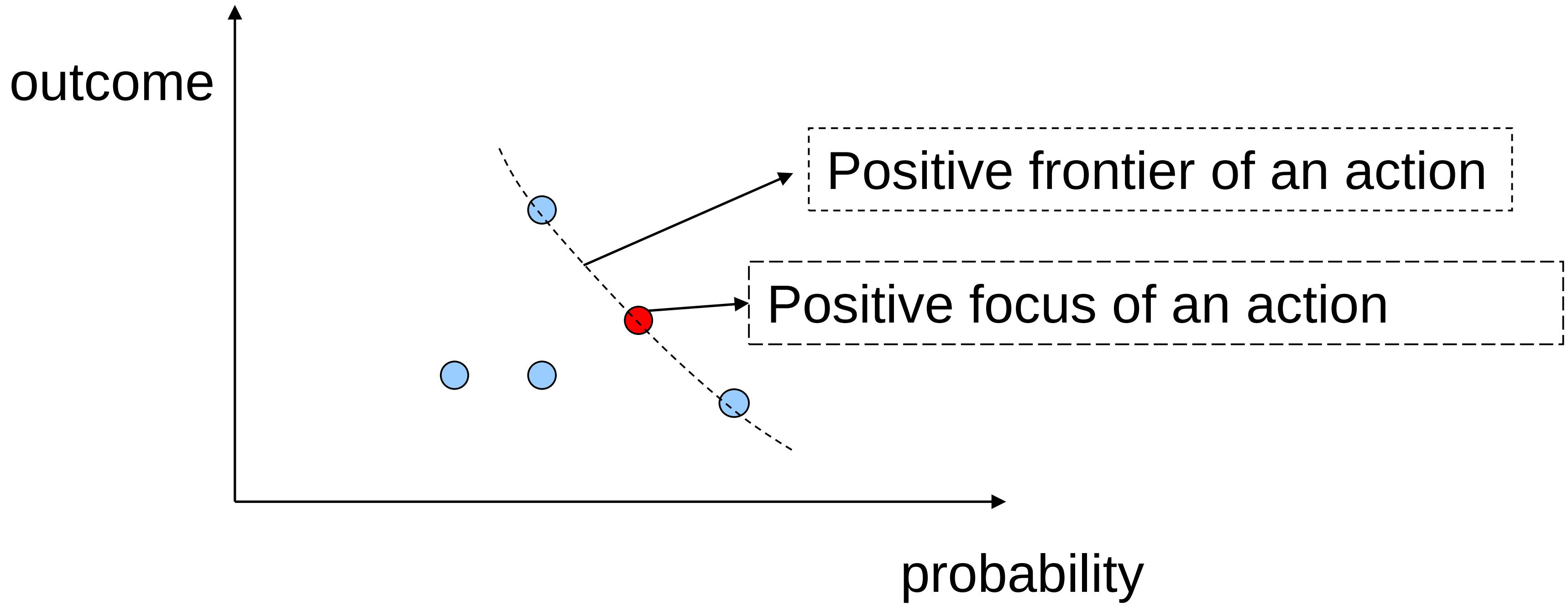
Axiom 2- Dominance:



Events of an action

Positive frontier of an action
Positive focus point of an action



Positive frontier of an action: the set of undominated events of an action.

Positive focus of an action: the most attractive event of an action.

The relative likelihood

If a function $\pi: S \to [0,1]$ satisfies $\max_{x \in S} \pi(x) = 1$,

then $\pi(x)$ is called a likelihood level function.

The relative likelihood function represents the relative position of likelihood.

$\pi(x) = P(x)/maxP(x)$

$P(x)$: the probability mass function

$\pi(x) = f(x)/maxf(x)$

$f(x)$: the probability density function

Satisfaction function

Satisfaction function: The normalized payoff function

representing the relative position of payoff.

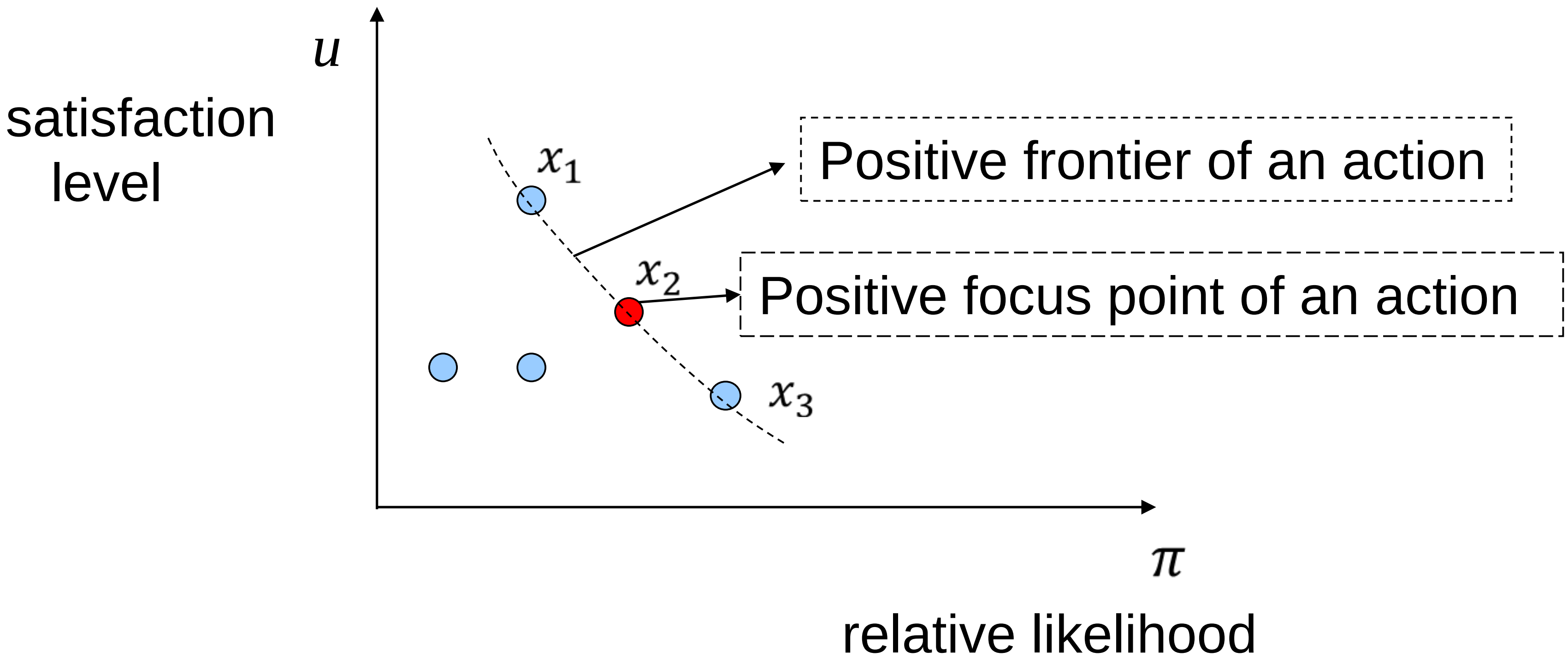
$u(x, a_i) = v(x, a_i)/\max v\,(x, a_i)$

$u(x, a_i)$: satisfaction function

$v(x, a_i)$: payoff function

Both relative likelihood function and satisfaction function are exogenously given.

Positive frontier of an action
Positive focus point of an action



Representation theorem of the positive focus point (Theorem 1)

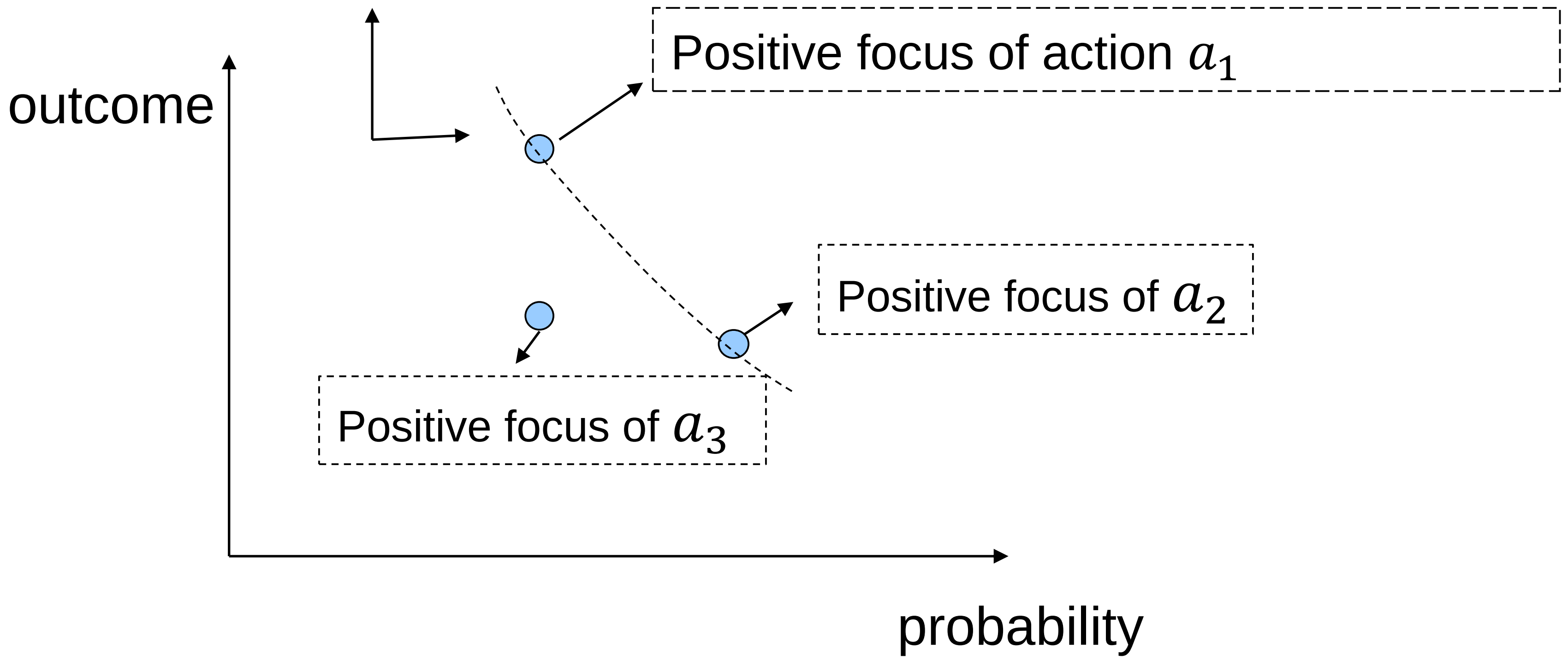
For x_2, there exists a function $\pi(x)\Lambda((1/\varphi) * u(x, a_i))$

$$\pi(x_2)\Lambda((1/\varphi) * u(x_2, a)) = max\begin{pmatrix}\pi(x_1)\Lambda\big((1/\varphi) * u(x_1, a)\big), \pi(x_2)\Lambda((1/\varphi) * u(x_2, a)) \\ , \pi(x_3)\Lambda((1/\varphi) * u(x_3, a))\end{pmatrix}$$

$\pi(x)\Lambda(1/\varphi) * u(x, a)$: the attractiveness of the focus x of action a

φ: the degree of emphasizing possible outcome for choosing the positive focus, endogenously derived from the choosing the positive focus of an action.

Dominance relationship between foci of two actions



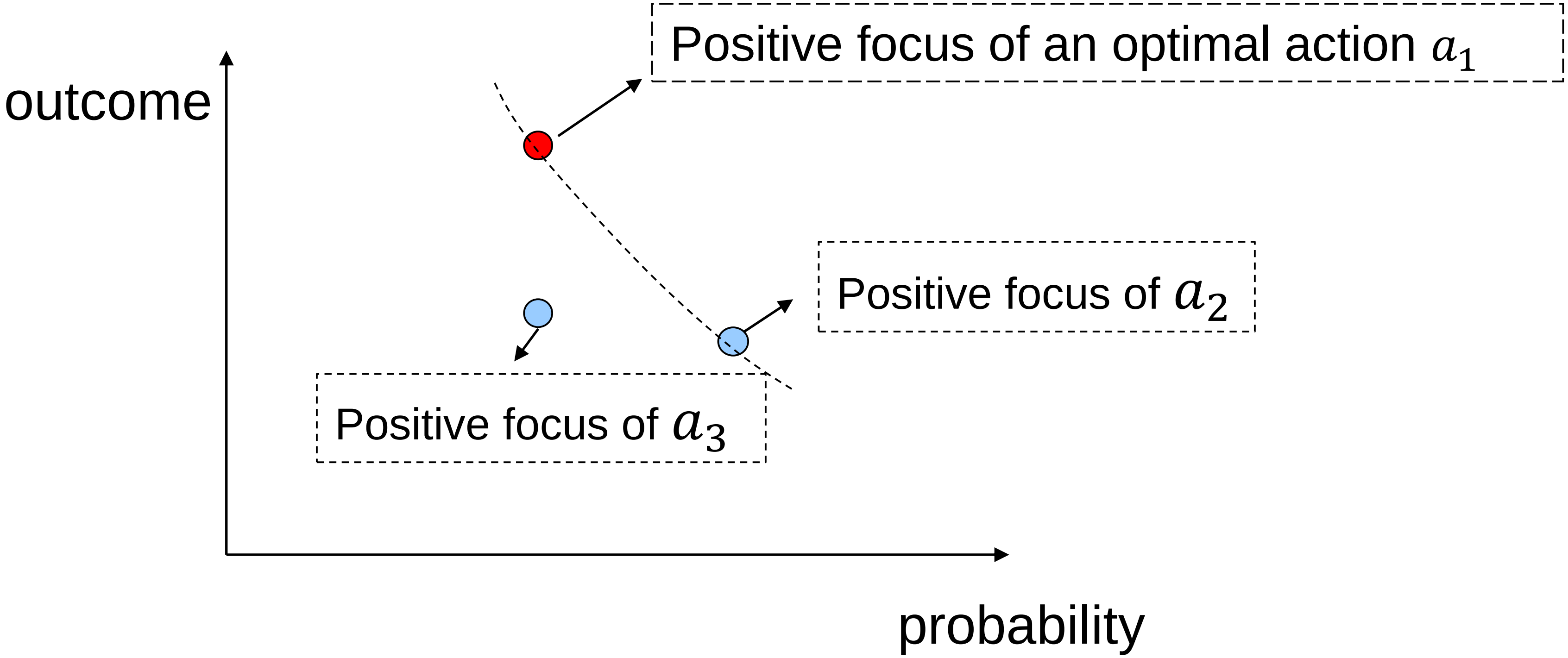
Axioms for the optimal action

Axiom 3- Decidability: A DM can choose the most preferred action.

It relaxes the assumptions of completeness and transitivity in standard economic theory and replace them by decidability. It means that a DM can determine his most-preferred action but there is no need to judge between any pair of actions. This assumption is intuitively appealing because in the real world the observable and observed action is usually the optimal action itself.

Axiom 4: Focus lexicographical dominance

Axiom 4 (1)



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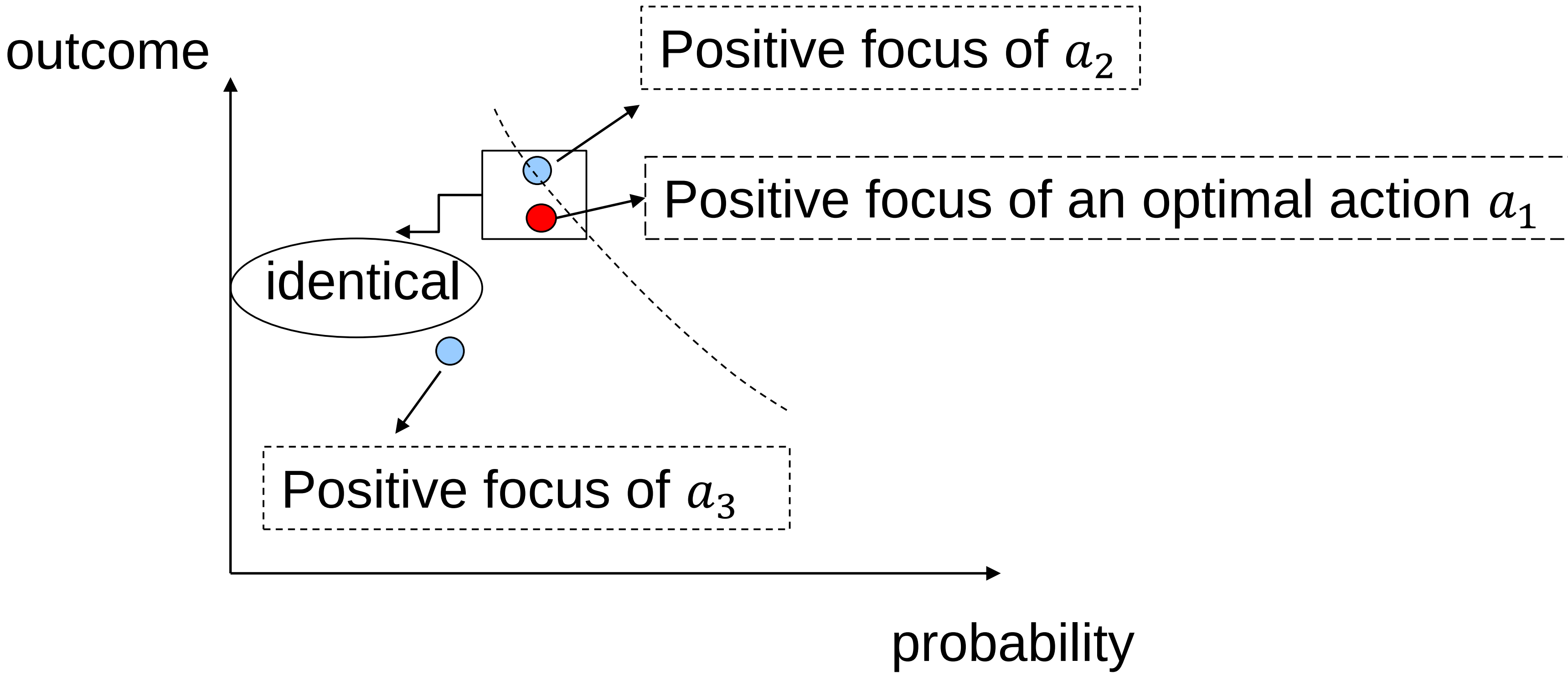
Axiom 4 (2)



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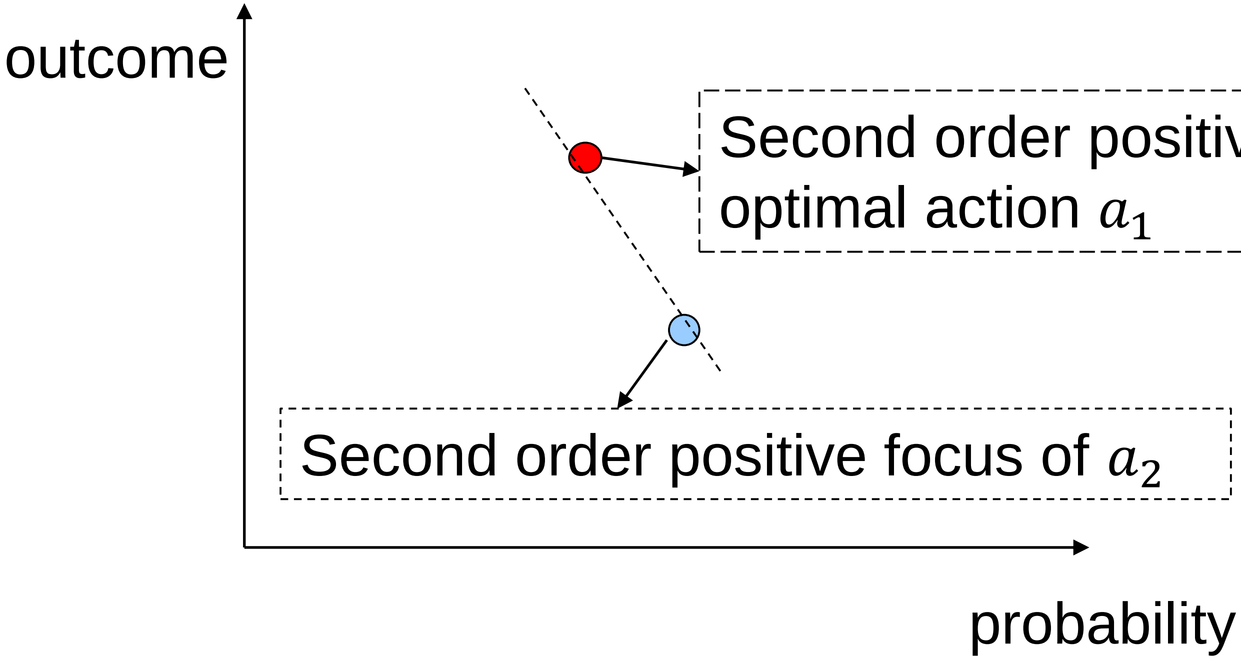
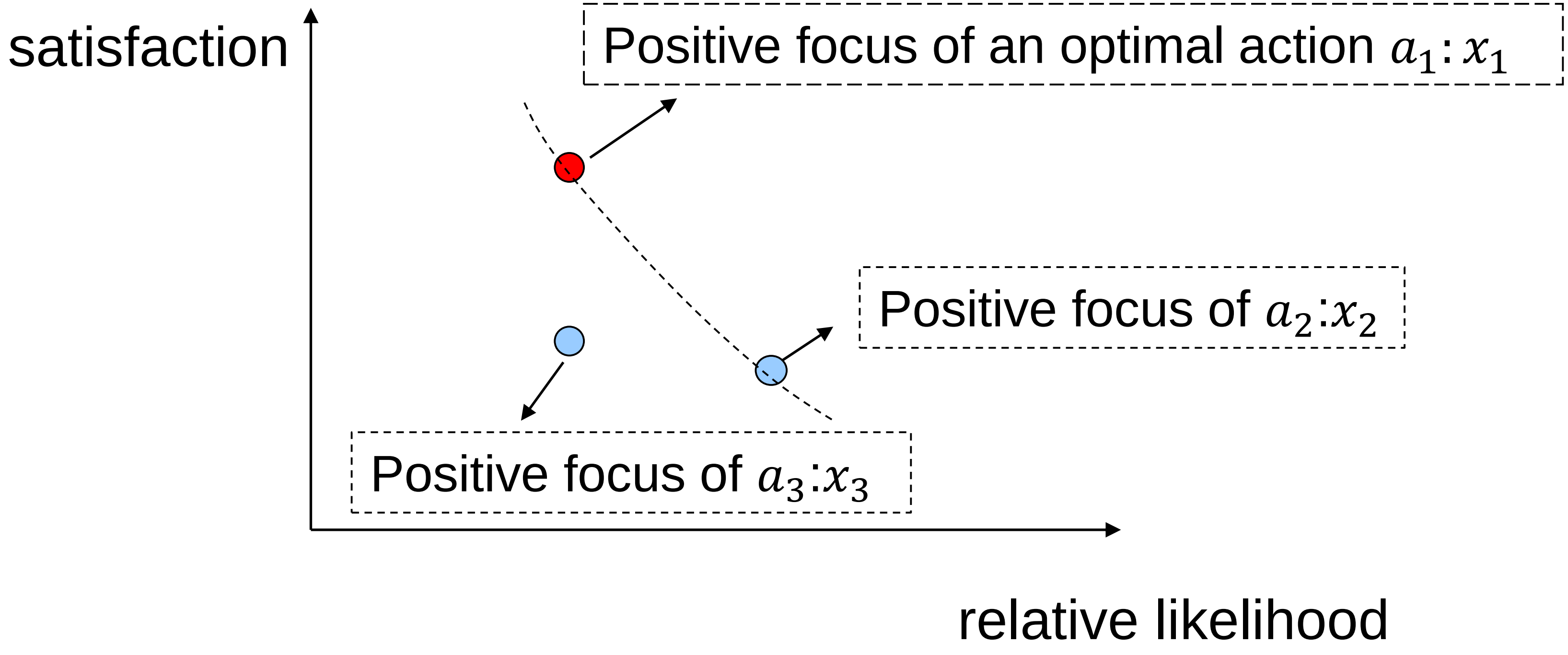
Axiom 4 (2)



Adjusted relative likelihood and satisfaction

Axiom 4 (1)



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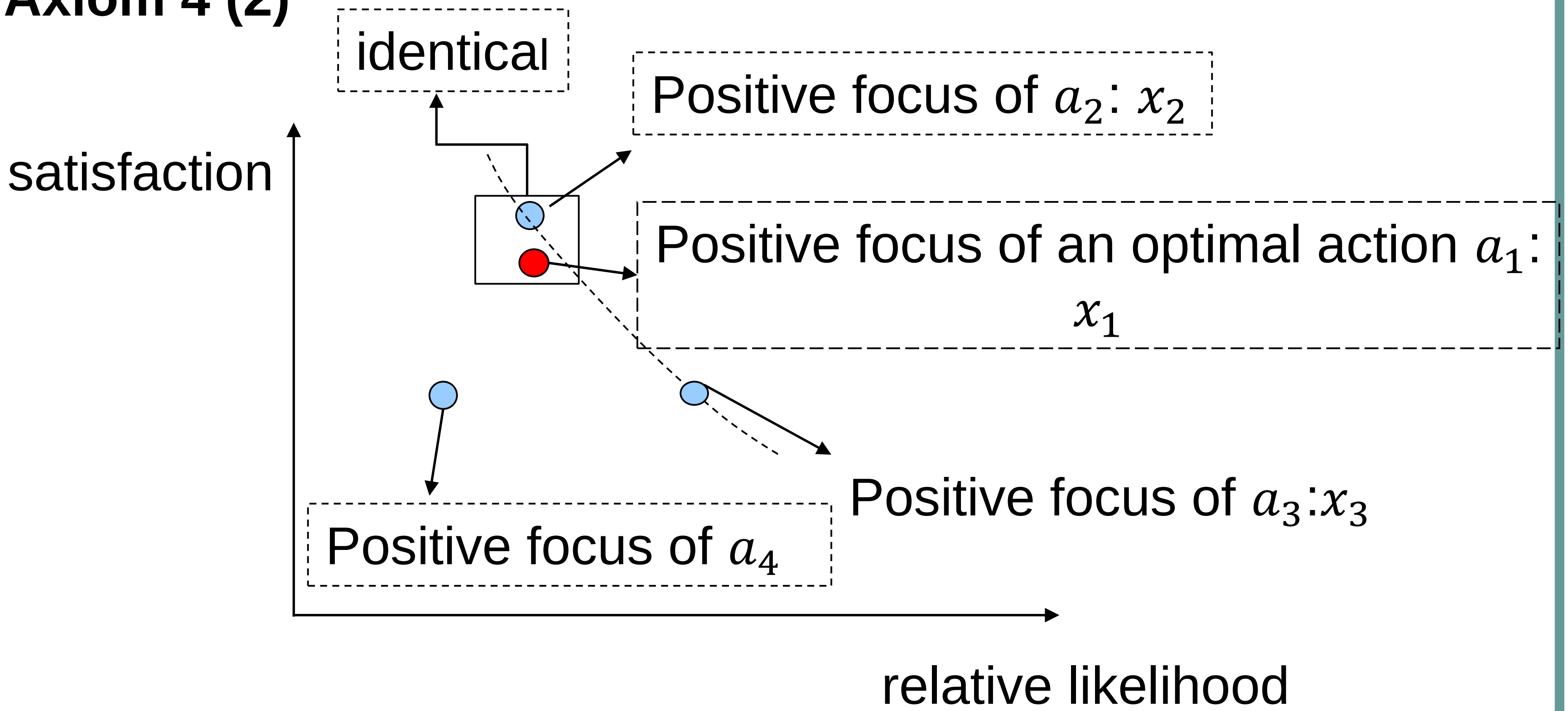
Axiom 4 (2)



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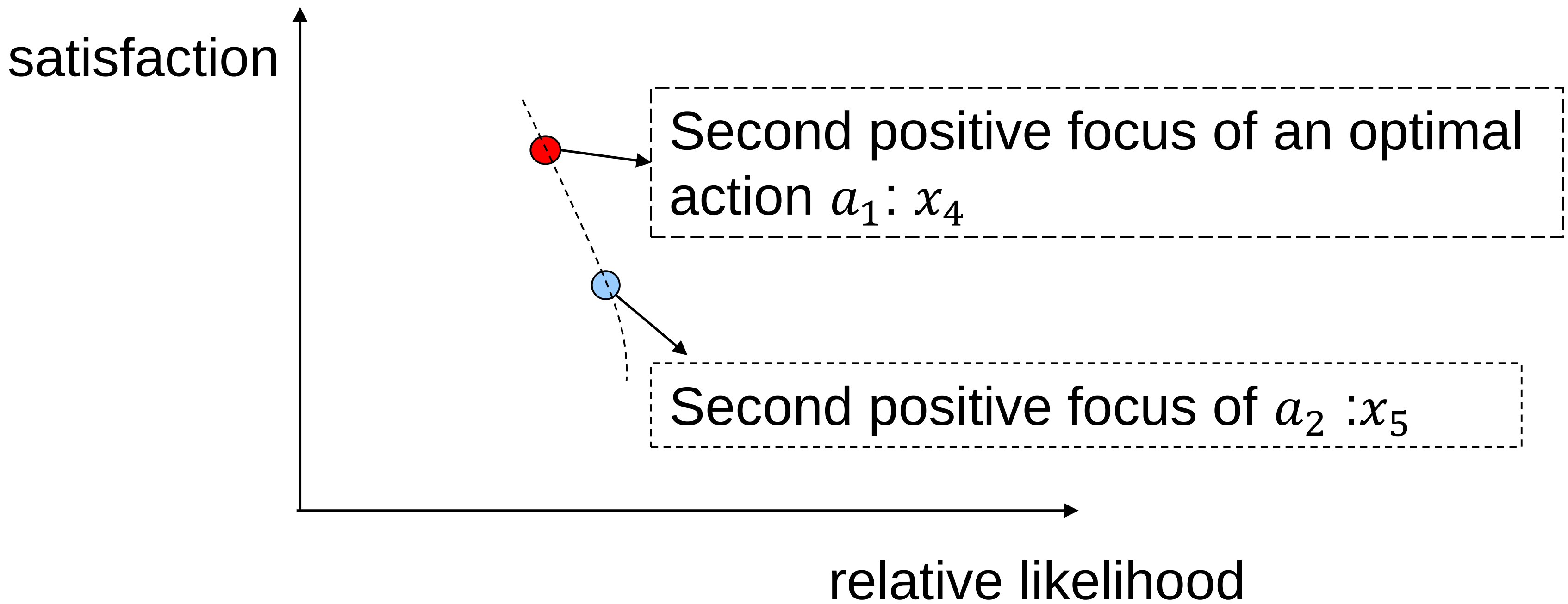
Axiom 4 (2)



Representation theorem of an optimal action (Theorem 2)

If the optimal action a_1 satisfies axiom 4 (1), then $\exists$ a function $\pi(x)\Lambda(1/\kappa) * u(x,a)$ satisfying

$$\pi(x_1)\Lambda((1/\kappa) * u(x_1, a_1)) =$$
$$max(\pi(x_1)\Lambda((1/\kappa) * u(x_1, a_1)), \pi(x_2)\Lambda((1/\kappa) * u(x_2, a_2)))$$

$min\left(\pi(x), (1/\kappa) * u(x, a)\right)$: the attractiveness of an action *a*

$\kappa > 0$: the degree of emphasizing the outcome, endogenously derived from choices of the optimal action.

$\kappa \uparrow \longrightarrow$ u is emphasized (pursuing possible effect)

$\kappa \downarrow \longrightarrow$ π is emphasized (pursuing certain effect)

Cont'd

If the optimal action satisfies axiom 4 (2), then

$\exists x_2$, $\exists \kappa$ satisfy

$$min\left(\pi(x_2),(1/\kappa)*u(x_2,a_2\ )\right)=$$

$$max\left(min\left(\pi(x_2),(1/\kappa)*u(x_2,a_2\ )\right),min\left(\pi(x_3),(1/\kappa)*u(x_3,a_3\ )\right)\right)$$

$$(x_1,x_2)\in=_\delta$$

$\exists x_4$, $\exists \kappa_{(1)}$ satisfy

$$min\left(\pi(x_4),\left(1/\kappa_{(1)}\right)*u(x_4,a_1\ )\right)=$$

$$max\left(min\left(\pi(x_4),\left(1/\kappa_{(1)}\right)*u(x_4,a_1\ )\right),min\left(\pi(x_5),\left(1/\kappa_{(1)}\right)*u(x_5,a_2\ )\right)\right)$$

Resolving the Ellsberg paradox

Two urns containing well mixed red and black balls.

Urn I contains exactly 5 red and 5 black balls.

Urn II contains 10 red and black balls, but in an entirely unknown ratio.

You have to choose an urn and draw a ball at random from it.

Please decide which you prefer, Urn I or Urn II in the following two gambles.

Gamble A: If you draw a red ball, you receive $100
and if you draw a black one, you receive nothing.

Gamble B: If you draw a black ball, you receive $100
and if you draw a red one, you receive nothing.

Empirical evidence shows that most subjects choose Urn I for both Gamble A and Gamble B; it violates Savage axioms.

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Existing theories for solving the Ellsberg paradox :

Hurwicz expected utility (Gul and Pesendorfer, 2015)

Expected uncertain utility theory (Gul and Pesendorfer, 2014)

Multiplier preference (Strzalecki, 2011)

Second-order probability models (Segal, 1987, Neilson, 2010)

Variational preference (Maccheroni, Marinacci and Rustichini, 2006)

Maximin expected utility (I. Gilboa and D. Schmeidler,1989)

Expected utility with nonadditivity measure (Schmeidler, 1989)

Analyzing with PES

The set of actions is {choosing Urn I, choosing Urn II}.

The set of events of choosing Urn I is {Black, Red}, and the positive focus of choosing Urn I is Red=(100, .5).

Since we do not know the ratio of red balls to black balls in Urn II, choosing Urn II is equivalent to a two-stage choosing as follows.

In Step I, choose one type of urn from 11 types of uniformly distributed urns. They are Type 1 {Black0, Red10}, Type 2 {Black1, Red9}, …, Type 11 {Black10, Red0} where Black1 means that there is 1 black ball in the urn while Red9 means that there are 9 red balls in the urn.

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In Step II, draw one ball from the chosen urn. Hence, the set of events for choosing Urn II is {B1, R1, B2, R2, …, B11, R11} where B1 represents drawing a black ball from a Type 1 Urn while R2 represents drawing a red ball from a Type 2 urn.

events	B1	R1	B2	R2	B3	R3	B4	R4	B5	R5	B6
probabilities	0	0.091	0.009	0.082	0.018	0.073	0.027	0.064	0.036	0.055	0.045
payoffs	0	100	0	100	0	100	0	100	0	100	0
events	R6	B7	R7	B8	R8	B9	R9	B10	R10	B11	R11
probabilities	0.045	0.055	0.036	0.064	0.027	0.073	0.018	0.082	0.009	0.091	0
payoffs	100	0	100	0	100	0	100	0	100	0	100

Since R=(100, 0.5) dominates R1=(100, 0.091), choosing Urn I is better than choosing Urn II. The same result can be obtained for Gamble B.

Resolving other anomalies

- The St. Petersburg paradox
- The Allais paradox
- Preference Reversal
- Event-Splitting Effect
- Violations of Tail-Separability
- Violations of Stochastic Dominance
- Violations of Transitivity
- Reflection effect
- Common ratio effect

The evidence supporting FTC

Stewart, Hermens, & Matthews, Journal of Behavioral Decision Making 29 (2016) 116-136.

“We found very little systematic variation in eye movements over the time course of a choice or across the different choices.

The only exceptions were finding more (of the same) eye movements when choice options were similar, and an emerging gaze bias in which people looked more at the gamble they ultimately chose.

Cont'd

The eye movements made during a choice have a large relationship with the final choice, and this is mostly independent from the contribution of the actual attribute values in the choice options.

Eye movements tell us not just about the processing of attribute values but also are independently associated with choice. The pattern is simple—people choose the gamble they look at more often, independently of the actual numbers they see".

Concluding remarks

1. We propose FTC which models and axiomatizes a decision-making procedure which are fundamentally different from the existing decision theories.
2. FTC provides a unified framework for handling decision making with risk or under ambiguity or under ignorance.
3. FTC can resolves many well-known anomalies, such as the St. Petersburg, Allais and Ellsberg paradoxes.
4. As a special case of FTC, one-shot decision theory has been applied to auction problems, newsvendor problems for innovative products, private real estate investment, multistage decision making problem, stochastic programming problems.
5. FTC provides a rigorous formal underpinning for behavioral economics and Behavioral operations research.

Thanks for your attendance