論　説

Strategic Uncertainty in Signaling Games with Multiple Priors

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Abstract

Two-player signaling games, where both players conceive uncertainty in the opponent's strategic choice, are examined. It is shown that equilibrium behavior largely depends on which update rule is adopted. Updated by the Dempster-Fagin-Halpern rule, we never suffer such unreasonable behavior on the equilibrium path as Ryan(2002) pointed out, and strategic uncertainty in hybrid or completely mixed equilibria is observed. It is also examined that, this is not due to the existence of uncertainty, or in a word, lack of confidence, but caused by the distinctive property of an update rule.

1 Introduction

Generalizing conventional solution concepts in games to conform to ambiguity or uncertainty averse behavior is a subject of many research interests in recent years. In a conventional setting of games, it is possible to incorporate various sources of uncertainty, such as opponents' strategic choices, private signals, the player set or the number of players, payoffs, or the game tree itself. In such games with uncertainty, a foundation for Nash equilibria would be weakened, since players may not have enough information or empirical trials to reduce uncertainty to risk.

The main object of this article is to examine two-player extensive games with imperfect information, where each player conceives uncertainty in the opponent's strategic choice.

As for complete information games in a traditional sense, Dow and Werlang(1994) extend the concept of a Nash equilibrium for two-player normal form games with uncertainty about opponent's choice. They also presented that, in the twice repeated prisoners' dilemma game, backward induction breaks down. In Eichberger and Kelsey(2000), the equilibrium notion of Dow and Werlang(1994) is generalized to n-player normal form games. Furthermore, some specific patterns of equilibrium behavior are characterized by means of a degree of

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confidence or ambiguity. Marinacchi(2000) extended the support for non-additive measures in Dow and Werlang(1994) and examined an equilibrium in beliefs (see Crawford(1990)).

With regard to the multiple priors model, Lo(1999) proposed multiple priors Nash equilibria in n-player finite extensive form games with complete information, where the sources of uncertainty are all other player's actions. As an update rule, the Dempster-Shafer rule (Shafer(1976)) and the full Bayesian update rule are examined.

In Kajii and Ui(2005), which addresses incomplete information games with multiple priors, players conceive ambiguity in signals they receive. As an update rule, the DS rule and the Dempster-Fagin-Halpern update rule (Dempster(1967), Fagin and Halpern(1991)) are incorporated, and the equilibrium notion based on mixed strategies and an equilibrium in beliefs are proposed.

Eichberger and Kelsey(2004), which is largely concerned with the games examined here, introduced a Dempster-Shafer equilibrium (DSE) that extends the notion of perfect Bayesian equilibria so that players' beliefs are represented by convex capacities with the DS updating. As a support of a convex capacity, a support of Dow and Werlang(1994) is adopted.

This paper introduces an equilibrium notion with multiple priors, which is represented by a convex capacity, which is called an equilibrium with multiple priors (EMP). There are three features of EMP adopted here.

One is that, in our multiple priors setting, every player's prior set is assumed to be represented by a convex capacity. In an equilibrium with multiple priors presented here, even if the lowest probability in some prior set is zero, the highest probability may not be zero, there is room for some mixed strategy contained in this range.

As the second characteristic, it is assumed that any prior set is represented by a general form of the convex capacity. Since convex capacities Eichberger and Kelsey(2004) adopted are symmetric, therefore the updated belief set is easily reduced to a singleton, especially via the DS rule. As illustrated in Section 4, it might conceal hybrid or completely mixed equilibria.

The last one is that, as an update rule, the Dempster-Fagin-Halpern rule is introduced. As Ryan(2002) pointed out, DSE has an interesting nature that, on the equilibrium path, some reasonable beliefs are discarded. It will be proved, in some simple examples, this never happens in EMP with the DFH rule. Therefore, the unreasonable behavior on the equilibrium path, may happen not due to the existence of uncertainty, or in a word, lack of confidence, it may be caused by the distinctive property of the update rule each player adopts.

This paper organized as follows: in Section 1, the signaling games and the basic definitions are presented. Section 2 illustrates the basic setting, and in the following section the notion of EMP is introduced and its existence is proved. Two simple signaling games are investigated in Section 4, and the properties of EMP are examined.

2　The Basic Model

We begin with a signaling game $G = [N, (S_i)_{i \in N}, (u_i)_{i \in N}, \mathcal{T}, \pi]$ defined in the following. Let

$N = \{1, 2\}$ be the set of players. At the beginning of the game, Nature chooses a type of player 1 from a finite set $\mathcal{T} = \{t_1, t_2, \ldots, t_n\}$, according to a probability distribution π on $\mathcal{T}$. Player 1's type is private information, that is, player 2 dose not know which type of player 1 is. Every player is to choose an action from A_i, a finite set of actions for each player $i = 1, 2$. Let $A = A_1 \times A_2$. For future reference, let K be the index set of A_1.

Let $\Delta(X)$ be the set of all probability distributions on a finite set X. The aforementioned π on $\mathcal{T}$ is a member of $\Delta(\mathcal{T})$. It is assumed that, throughout this paper, this π is commonly known.

Player 1 may choose a type-dependent mixed action, so player 1's strategy is a function σ_1 from $\mathcal{T}$ to $\Delta(A_1)$. Let S_1 be the set of such strategies for player 1. Denote the probability distribution on $\mathcal{T} \times A_1$ induced by π and σ_1, by $p(\pi, \sigma_1)$, which is a member of $\Delta(\mathcal{T} \times A_1)$. On the other hand, player 2 chooses the action from A_2 after observed the action of player 1. Player 2's strategy is a function from the observed action to a mixed action on A_2, i.e. a function σ_2 from A_1 to $\Delta(A_2)$. Let S_2 be the set of all strategies of player 2.

As usual, let $u_i : A \times \mathcal{T} \to \mathbb{R}$, $i = 1, 2$ represents player i's payoff from any combination of actions of two players and a type of player 1. For every type t of player 1, the expected utility is denoted by

$$U_1(\sigma_1^t, \sigma_2, t) = \sum_{k \in K} \sum_{a_2 \in A_2} \sigma_1^t(a_1^k) \cdot \sigma_2^k(a_2) \cdot u_1(a_1^k, a_2, t).$$

It will be also useful below, the interim expected utility of every type t choosing a_1^k is

$$U_1(a_1^k, \sigma_2, t) = \sum_{a_2 \in A_2} \sigma_2^k(a_2) \cdot u_1(a_1^k, a_2, t).$$

A belief of player 2 after a_1^k is observed, is expressed by $\rho \in \Delta(\mathcal{T})$. Given ρ, player 2's expected utility is denoted by

$$U_2(a_1^k, \sigma_2^k) = \sum_{t \in \mathcal{T}} \sum_{a_2 \in A_2} \rho(t \mid a_1^k) \cdot \sigma_2^k(a_2) \cdot u_2(a_1^k, a_2, t).$$

The conventional equilibrium notion of G is a (weak) perfect Bayesian equilibrium, defined as follows.

Definition 1 *A perfect Bayesian equilibrium (PBE) of G is a set of strategies* $[\sigma_1^*, \sigma_2^*] = \left[(\sigma_1^{t\,*})_{t \in \mathcal{T}}, (\sigma_2^{k\,*})_{k \in K}\right]$ *and ρ satisfying:*

(PB-1) player 1 of every type t in $\mathcal{T}$ chooses $\sigma_1^{t\,*}$ such that

$$\sigma_1^{t\,*} \in \arg\max_{\sigma_1^t \in \Delta(A_1)} U_1\left(\sigma_1^t, \sigma_2^*, t\right).$$

(PB-2) player 2 who observed every a_1^k in A_1 chooses σ_2^k such that, given a ρ

$$\sigma_2^{k*} \in arg \max_{\sigma_2^k \in \Delta(A_2)} U_2\left(a_1^k, \sigma_2^k\right).$$

(PB-3) For every a_1^k in A_1,

$$\rho\left(t \mid a_1^k\right) = \frac{\sigma_1^{t*}(a_1^k)}{\sum_{\tau \in \mathcal{T}} \sigma_1^{\tau*}(a_1^k)} \text{ if } \sum_{\tau \in T} \sigma_1^{\tau*}\left(a_1^k\right) > 0.$$

3　Equilibrium with Multiple Priors

Now we assume that a player's subjective ambiguous situation is summarized in the prior set. In view of multiple-prior modelling where players confront subjective uncertainty about the opponent's mixed choice, it is assumed that the belief every player has is represented as an set of beliefs which may not be a singleton.

In our setting of games, player 1 adopts a belief after played an action a_1^k in A_1 about player 2's behavior. Player 1's belief is assumed to take the form of a non-empty, compact set of probability measures on A_2. More specifically, denote this player 1's belief set for each $k \in K$ by $\mathcal{P}_1^k \subset \Delta(A_2)$. As for player 2, priors about player 1's types and behavior is a compact set of probability measures on $\mathcal{T} \times A_1$, i.e. $\mathcal{P}_2 \subset \Delta(\mathcal{T} \times A_1)$.

To describe the ambiguous situation, we introduce non-additive measures, especially convex capacities. Let Ω be the finite set of states and Σ is an algebra, $\Sigma = 2^{\Omega}$. A capacity on Σ is a set function μ: $\Sigma \to [0,1]$ satisfying (i) $\mu(\emptyset) = 0$ and $\mu(\Omega) = 1$, and (ii) monotonicity: for every S and T in Σ such that $S \subset T$, we have $\mu(S) \leq \mu(T)$. A capacity μ is convex if for any S and T in Σ, $\mu(S \cup T) + \mu(S \cap T) \geq \mu(S) + \mu(T)$. $\overline{\mu}$ is called the conjugate of μ, which is defined as $\overline{\mu}(S) \equiv 1 - \mu(\Omega \setminus S)$. Throughout this paper, it is assume that μ is a convex capacity.

Definition 2 *$\mathcal{P}$ is represented by μ if*

$$\mathcal{P} = \{p \in \Delta(X) \mid p(S) \geqq \mu(S) \text{ for all } S \subset X\}.$$

When $\mathcal{P}$ is represented by μ (i.e. the core of a convex capacity μ), it is written as $\mathcal{P}(\mu)$. In our setting of games, let μ_1^k be defined on 2^{A_2} for every $k \in K$ and μ_2 be on $2^{\mathcal{T} \times A_1}$. Then, we write $\mathcal{P}_1^k$, $\mathcal{P}_2$ *represented by* μ_1^k, μ_2 as $\mathcal{P}\left(\mu_1^k\right)$, $\mathcal{P}\left(\mu_2\right)$, respectively.

When player 2's prior set $\mathcal{P}_2$ is given and a_1^k was observed, the prior set $\mathcal{P}_2$ is to be revised according to this information, say $\mathcal{P}_2^k \subset \Delta(\mathcal{T})$. Let Φ express an update rule that transforms $\left(\mathcal{P}_2 \middle| a_1^k\right)$ into $\mathcal{P}_2^k$. It is assumed that $\mathcal{P}_2$ is represented by a convex capacity μ_2, so $\mathcal{P}_2^k$ is represented by a convex capacity μ_2^k on $2^{\mathcal{T}}$. Then Φ is characterized by the update rule for μ_2, that is, $\left(\mathcal{P}(\mu_2) \middle| a_1^k\right) \longmapsto \Phi\left(\mathcal{P}(\mu_2) \middle| a_1^k\right) = \mathcal{P}\left(\mu_2^k\right)$, which is also assumed throughout this paper.

Now the specification of G in the multiple-priors version is denoted by $\mathcal{G}$,

$\mathcal{G} = \left[N, (S_i)_{i=1,2}, (u_i)_{i=1,2}, \mathcal{T}, \pi, \Phi\right]$. The following defines an equilibrium with multiple priors.

Definition 3 *An equilibrium with multiple priors (EMP) of* $\mathcal{G}$ *is a set of strategies* $[\sigma_1^*, \sigma_2^*]$ *and prior sets* $\left[(\mathcal{P}_1^k)_{k \in K}, \mathcal{P}_2\right]$ *satisfying the following conditions:*

(MP-1) $\mathcal{P}_1^k \subset \Delta(A_2)$ *and* $\mathcal{P}_1^k = \mathcal{P}(\mu_1^k)$ *for every* a_1^k *in* A_1, *and* $\mathcal{P}_2 \subset \Delta(\mathcal{T} \times A_1)$ *and* $\mathcal{P}_2 = \mathcal{P}(\mu_2)$.

(MP-2) For every $a_1^k \in A_1$, $\mathcal{P}_2^k \subset \Delta(\mathcal{T})$ *and* $\mathcal{P}_2^k = \mathcal{P}(\mu_2^k)$ *where* $\mathcal{P}_2^k$ *is the posterior set updated by* Φ.

(MP-3) Player 1 of every type t in $\mathcal{T}$ *chooses* $\sigma_1^{t\,*}$ *such that if* $\sigma_1^{t\,*}(a_1^k) > 0$, *then*

$$a_1^k \in arg \max_{a_1 \in A_1} \left[\min_{\sigma_2 \in \mathcal{P}(\mu_1^k)} U_1(a_1, \sigma_2, t) \right].$$

(MP-4) If player 2 who observed every a_1^k *in* A_1 *chooses* $\sigma_2^{k\,*}$ *such that*

$$\sigma_2^{k\,*} \in arg \max_{\sigma_2 \in \Delta(A_2)} \left[\min_{\rho \in \mathcal{P}(\mu_2^k)} U_2(a_1^k, \sigma_2) \right],$$

(MP-5) $p(\pi, \sigma_1^*) \in \mathcal{P}(\mu_2)$ *and* $\sigma_2^{k\,*} \in \mathcal{P}(\mu_1^k)$ *for any* $k \in K$.

Definition 4 *An EMP agrees with the common prior* π *iff for every* $T \subset \mathcal{T}$,

$$\mu_2(T \times A_1) = \overline{\mu}_2(T \times A_1) = \pi(T).$$

This condition is proposed by Eichberger and Kelsey(2004), μ_2's consistency with the commonly known π.

Theorem 1 *There exists an equilibrium with multiple priors of* $\mathcal{G}$ *which agrees with the common prior* π.

Proof At first, pick μ_1^k, μ_2 specified as follows: for any $k \in K$,

$$\mu_1^k(S) = \begin{cases} 0 & \text{if } S \neq A_2 \\ 1 & \text{if } S = A_2 \end{cases} \quad \text{for every } S \subset A_2.$$

For every $S \subset \mathcal{T} \times A_1$

$$\mu_2(S) = \begin{cases} 0 & \text{if } S \cap (T \times A_1) = \varnothing \text{ for any } T \subset \mathcal{T} \\ \pi(T) & \text{if } T \text{ is the largest subset of } \mathcal{T} \text{ such that } S \cap (T \times A_1) = T \times A_1 \end{cases}$$

Obviously, every μ_1^k and μ_2 are convex capacities. Hence each $\mathcal{P}(\mu_1^k)$, $\mathcal{P}(\mu_2)$ is a non-empty, compact, and convex subset of $\Delta(A_2)$, $\Delta(\mathcal{T} \times A_1)$ respectively.

When every type t of player 1 chooses to play $a_1^k \in A_1$, every type t has an probabilistic assessment that player 2 reacts by choosing some mixed action $\sigma_2^k \in \mathcal{P}(\mu_1^k)$, the type t's interim payoff is:

$$\min_{\sigma_2 \in \mathcal{P}(\mu_1^k)} U_1\left(a_1^k, \sigma_2, t\right).$$

Let $\mu_1 = (\mu_1^1, \ldots, \mu_1^K)$ and $B_1^t(\mu_1)$ be the best response for type t defined as

$$B_1^t(\mu_1) = arg \max_{\sigma_1 \in \Delta(A_1)} \sum_{k \in K} \sigma_1^t\left(a_1^k\right)\left[\min_{\sigma_2 \in \mathcal{P}(\mu_1^k)} U_1(a_1^k, \sigma_2, t)\right].$$

Since each $U_1\left(a_1^k, \sigma_2, t\right)$ is continuous in σ_2 and $\mathcal{P}\left(\mu_1^k\right)$ is a non-empty, compact, and convex subset of $\Delta(A_2)$ and continuous in μ_1^k, its minimum over $\mathcal{P}\left(\mu_1^k\right)$ is also continuous and concave in μ_1^k. Therefore $B_1^t(\mu_1)$ is non-empty, compact and convex valued and upper hemi-continuous.

Player 1's best response is denoted by

$$B_1(\mu_1) = \left\{p \in \Delta(\mathcal{T} \times A_1) |\; p\left(t, a_1^k\right) = \pi(\{t\}) \cdot \sigma_1^t\left(a_1^k\right), \sigma_1^t \in B_1^t(\mu_1)\right\}.$$

$B_1(\mu_1)$ is also non-empty, compact and convex valued, and upper hemi-continuous, as $B_1^t(\mu_1)$ is defined above and $\pi \in \Delta(\mathcal{T})$ is given.

Player 2 considers that, when a_1^k in A_1 is observed, the posterior set $\mathcal{P}(\mu_2^k)$ is represented by some updated convex capacity μ_2^k on $\mathcal{T}$ according to Φ, therefore $\mathcal{P}(\mu_2^k)$ is a non-empty, compact, and convex subset of $\Delta(\mathcal{T})$ Now player 2's best response is written as

$$B_2\left(\mu_2^k\right) = arg \max_{\sigma_2 \in \Delta(A_2)}\left[\min_{\rho \in \mathcal{P}(\mu_2^k)} U_2(a_1^k, \sigma_2)\right],$$

which is also non-empty, upper hemi-continuous, and convex valued.

Now consider a mapping $\Theta: \Delta(\mathcal{T} \times A_1) \times \Delta(A_2)^K \longmapsto \Delta(\mathcal{T} \times A_1) \times \Delta(A_2)^K$ such that

$$(\sigma_1, \sigma_2) \longmapsto \Theta(\sigma_1, \sigma_2) \equiv B_1(\mu_1) \times \left[\prod_{k \in K} B_2\left(\mu_2^k\right)\right].$$

$\Delta(\mathcal{T} \times A_1) \times \Delta(A_2)^K$ is compact and convex, since $\Delta(\mathcal{T} \times A_1)$ and $\Delta(A_2)$ are compact and convex. Θ is nonempty, compact and convex valued, and upper hemi-continuous, since $B_1(\mu_1)$ and every $B_2(\mu_2^k)$ are. Therefore, applying Kakutani's fixed point theorem, there exists a $(\sigma_1^*, \sigma_2^*) \in \Theta(\sigma_1^*, \sigma_2^*)$. Clearly, $p(\pi, \sigma_1^*) \in \mathcal{P}(\mu_2)$ and $\sigma_2^{k*} \in \mathcal{P}(\mu_1^k)$ for any $k \in K$. □

There are various conditioning rules for a convex capacity, however, among them, we particularly concentrate on two update rules, the Dempster-Shafer rule (DS rule) and the Dempster-Fagin-Halpern rule (DFH rule).

Definition 5 *For any event $E \in \Sigma$ such that $\mu(E^c) < 1$, the Dempster-Shafer update of μ (the*

DS rule) E is

$$\mu_E(S) = \frac{\mu(S \cup E^c) - \mu(E^c)}{1 - \mu(E^c)} \text{ for every } S \subset E.$$

The Dempster-Fagin-Halpern rule is defined as follows.

Definition 6 *For any event $E \in \Sigma$ such that $\overline{\mu}(E) > 0$, the Dempster-Fagin-Halpern update of μ (the DFH rule) conditional on E is*

$$\mu_E(S) = \frac{\mu(S)}{\mu(S) + \overline{\mu}(E \setminus S)} \text{ for every } S \subset E.$$

It is well known that both DS and DFH rules preserves monotonicity and convexity.

In our context, the set of states is $\mathcal{T} \times A_1$, and player 2's capacity is revised after observing player 1's action a_1^k, $k \in K$, i.e. event $\mathcal{T} \times \{a_1^k\}$ was observed. The updated capacity conditional on $\mathcal{T} \times \{a_1^k\}$ is denoted by μ_2^k which is replaced by μ_E above, and so μ_2^k is on $2^{\mathcal{T}}$.

Let g^{DS}, g^{DFH} be the game played by using the DS rule and DFH rule, respectively.

The following corollary illustrates that a perfect Bayesian equilibrium is also an equilibrium of EMP in g^{DS} or g^{DFH}.

Corollary 2 *There exists an equilibrium with multiple priors of g^{DS} or g^{DFH} which agrees with the common prior π.*

Proof. It is easily verified that, for every game g^{DS} or g^{DFH}, a perfect Bayesian equilibrium is correspond to the case where $\mu_1 = p\left(\pi, \sigma_1^*\right)$ and $\mu_2^k = \sigma_2^{k*}$ for every $k \in K$, and both update rules are conventional Bayes rule if each capacity is additive (a probability measure). Hence, from the definition of a EMP, a perfect Bayesian equilibrium is a EMP which is agree with the common prior π. □

4 Simple Examples

In this section, let us examine two simple signaling games with two types and actions.

4.1 Example with Strictly Dominant Strategy

In Eichberger and Kelsey(2004), the Dempster-Shafer equilibria (DSE) of some signaling games are illustrated, and Ryan(2002) is discussing some properties of DSE in view of belief persistence. Figure 1 is a typical example where, after the dominating action of player 1, player 2's updated belief seems to abandon some reasonable beliefs.

It is easily verified that this game has the separating perfect Bayesian equilibrium where player 1 of type t_1 plays L and type t_2 plays R, and player 2 always chooses U.

Eichberger and Kelsey(2004) show that, under a high level of ambiguity, there exists a set of beliefs which agrees with the prior distribution $(\pi(t_1), \pi(t_2)) = (0.5, 0.5)$ and constitutes a

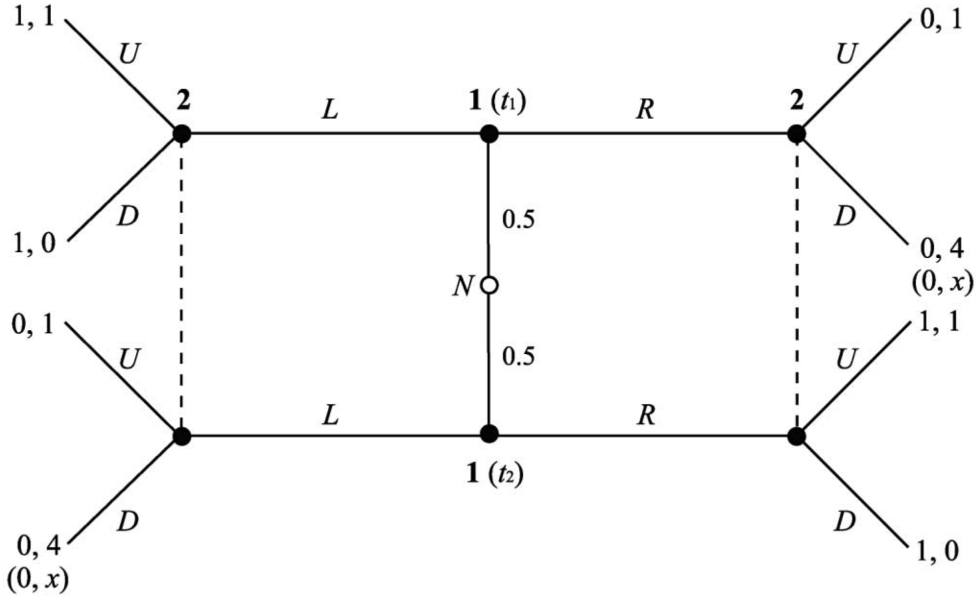


Figure1: An example in Eichberger and Kelsey(2004) and Ryan(2002).

DSE such that player 2 chooses D in response to L and R.

EMP with the DS rule also includes $\{(L,R),(D,D)\}$, however, using the DFH rule, there is no EMP other than $\{(L,R),(U,U)\}$. It suggests that the emergence of $\{(L,R),(D,D)\}$ as an EMP is not due to the degree of ambiguity, but due to the property of the update rule employed.

The following claim states that, it does not depend on the magnitude of payoff x shown in Figure 1.

Claim 1 *In the example in Figure 1, for any* $x > 1$*, there is no EMP with the DFH rule other than* $\{(L,R),(U,U)\}$.

Proof. Suppose that player 2's prior capacity μ_2 is given as following:

$$\mu_2(\{(t_1,L)\}) = \frac{\alpha_1}{2}, \mu_2(\{(t_1,R)\}) = \frac{\alpha_2}{2}$$
$$\mu_2(\{(t_2,L)\}) = \frac{\beta_1}{2}, \mu_2(\{(t_2,R)\}) = \frac{\beta_2}{2}$$

$$\mu_2(\{(t_1,L),(t_1,R)\}) = \frac{1}{2}, \mu_2(\{(t_2,L),(t_2,R)\}) = \frac{1}{2}$$

$$\mu_2(\{(t_1,L),(t_1,R),(t_2,R)\}) = \frac{1+\beta_3}{2}, \mu_2(\{(t_1,R),(t_2,L),(t_2,R)\}) = \frac{1+\alpha_3}{2}$$
$$\mu_2(\{(t_1,L),(t_1,R),(t_2,L)\}) = \frac{1+\beta_4}{2}, \mu_2(\{(t_1,L),(t_2,L),(t_2,R)\}) = \frac{1+\alpha_4}{2}$$
$$\mu_2(\mathcal{T} \times A_1) = 1.$$

This μ_2 agrees with π. Convexity of μ_2 requires that

$$\begin{aligned} \alpha_1+\alpha_2 &\leq 1, \beta_1+\beta_2 \leq 1 \\ \alpha_3 &\geq \alpha_2, \beta_3 \geq \beta_2 \\ \alpha_4 &\geq \alpha_1, \beta_4 \geq \beta_1. \end{aligned}$$

When player 2 observes L is played, DFH update for μ_2 yields

$$\begin{aligned} \mu_2^L(t_1) &= \frac{\mu_2(\{(t_1,L)\})}{\mu_2(\{(t_1,L)\})+1-\mu_2(\{(t_1,L),(t_1,R),(t_2,R)\})} \\ &= \frac{\alpha_1/2}{\alpha_1/2+1-\frac{1+\beta_3}{2}} = \frac{\alpha_1}{\alpha_1+1-\beta_3} \\ \mu_2^L(t_2) &= \frac{\mu_2(\{(t_2,L)\})}{\mu_2(\{(t_2,L)\})+1-\mu_2(\{(t_1,R),(t_2,L),(t_2,R)\})} \\ &= \frac{\beta_1/2}{\beta_1/2+1-\frac{1+\alpha_3}{2}} = \frac{\beta_1}{\beta_1+1-\alpha_3} \end{aligned}$$

Playing U gives player 2 the certain payoff 1, and playing D yields 0 if player 1 is type t_1 and x if type t_2, therefore player 2's minimum payoff will be $\frac{x\beta_1}{\beta_1+1-\alpha_3}$. Playing D is the best response if

$$\begin{aligned} \frac{x\beta_1}{\beta_1+1-\alpha_3} &\geq 1, \quad \text{or} \\ \beta_1 &\geq \frac{1-\alpha_3}{x-1} \end{aligned} \tag{1}$$

On the other hand, suppose that R is observed. In this case,

$$\begin{aligned} \mu_2^R(t_1) &= \frac{\mu_1(\{(t_1,R)\})}{\mu_1(\{(t_1,R)\})+1-\mu_1(\{(t_1,L),(t_1,R),(t_2,L)\})} \\ &= \frac{\alpha_2/2}{\alpha_2/2+1-\frac{1+\beta_4}{2}} = \frac{\alpha_2}{\alpha_2+1-\beta_4} \\ \mu_2^R(t_2) &= \frac{\mu_1(\{(t_2,R)\})}{\mu_1(\{(t_2,R)\})+1-\mu_1(\{(t_1,L),(t_2,L),(t_2,R)\})} \\ &= \frac{\beta_2/2}{\beta_2/2+1-\frac{1+\alpha_4}{2}} = \frac{\beta_2}{\beta_2+1-\alpha_4}. \end{aligned}$$

Playing U gives player 2 the certain payoff 1, and playing D yields x if player 1 is type t_1 and

0 if type t_2, therefore player 2's minimum payoff will be $\frac{x\alpha_2}{\alpha_2+1-\beta_4}$. Playing D is the best response if

$$\begin{aligned} \frac{x\alpha_2}{\alpha_2+1-\beta_4} &\geq 1, \text{or} \\ \beta_4 &\geq -(x-1)\alpha_2+1 \end{aligned} \tag{2}$$

Consider player 1's choice. Playing L for t_1 and R for t_2 are strictly dominant strategies, Playing (L,R) (pure actions) should be included in player 2's prior set, i.e.,

$$\begin{aligned} \overline{\mu}_2(\{(t_1,L)\}) &= \frac{1}{2}, \quad \mu_2(\{(t_1,R)\}) = 0 \\ \mu_2(\{(t_2,L)\}) &= 0, \quad \overline{\mu}_2(\{(t_2,R)\}) = \frac{1}{2}, \end{aligned}$$

hence $\alpha_2 = \alpha_3 = 0$ and $\beta_1 = \beta_4 = 0$. However, to have D as the best response, (1) and (2) requires that $\beta_1 \geq 1/(x-1)$ and $\beta_4 = 1$, so (D,D) cannot be supported as an EMP. This result is independent of the magnitude of x. □

4.2 Example of Hybrid Equilibrium

Let us now consider the following signaling game in Figure 2.

This game has a pooling and a separating perfect Bayesian equilibria:

$$\text{(S)} \quad \begin{cases} \text{player 1: type } t_1 \text{ plays } R \text{ and type } t_2 \text{ plays } L \\ \text{player 2: plays } U \text{ in response to } L \text{ and } R \end{cases}$$

$$\text{(P)} \quad \begin{cases} \text{player 1: both types plays } L \\ \text{player 2: plays } U \text{ in response to } L \text{ and plays } D \text{ in response } R \\ \qquad\qquad \text{with any belief } \rho(t_1|R) \leq 2/3 \end{cases}$$

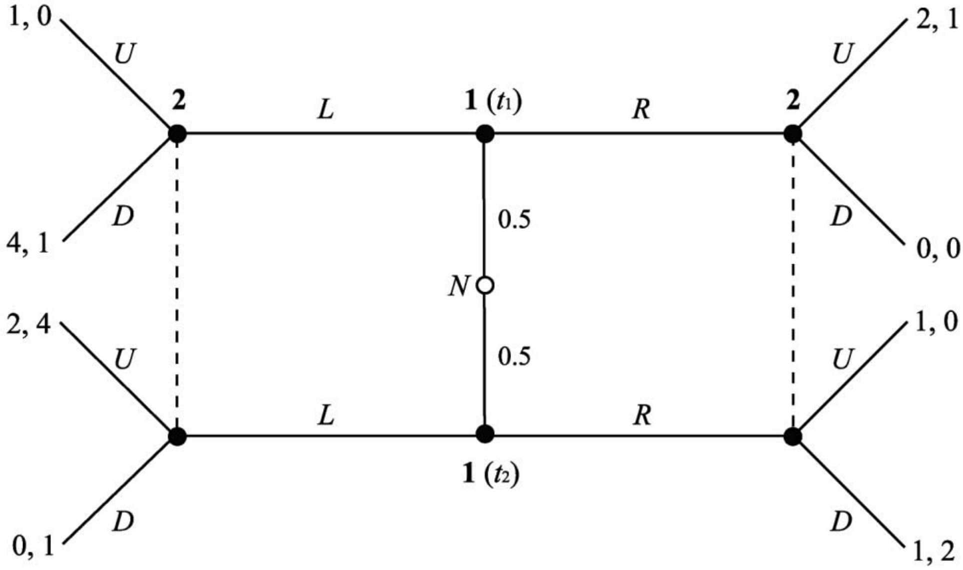
4.2.1 EMP with the DS rule

It is well known that a posterior set updated by the DS rule is smaller than the DFH rule (Denneberg(1994)). Especially, a two-states symmetric capacity revised by the DS rule becomes additive. In this case, any hybrid EMP might disappear. This is easily verified as follows. Suppose that, type t_1 chooses a mixed action and type t_2 chooses L. When R is observed, player 2 believes that the player 1 is t_1 correctly, hence player 2 never chooses mixed action. However it dose not happen in EMP with the DFH rule.

4.2.2 EMP with the DFH rule

Completely mixed equilibrium With the DFH rule, the game in Figure 2 has a completely mixed equilibrium if every type of player 1 is also uncertainty averse player who has non-degenerated prior set.

To see this, suppose that every type of player 1 is the expected utility maximizer and mixing the actions. Then, both types are indifferent between L and R, so player 2 plays U with



probability of p_L after observing L, and plays D with probability of p_R after R. For type t_2 to be indifferent between L and R, we have to have $p_L = 1/2$, however the condition for type t_1 is that $p_L + 4(1 - p_L) = 2p_R$. $p_L = 1/2$ cannot satisfy this requirement, hence there is no completely mixed equilibrium in this case.

The uncertainty averse behavior of player 1 drastically alternates this situation. Now suppose that player 1 is also uncertainty averse. Player 1's belief sets are denoted by $\mathcal{P}(\mu_1^a)$, $a = L, R$:

$$\begin{aligned} \mu_1^L(U) &= a_1, \mu_1^L(D) = b_1, \\ \mu_1^R(U) &= a_2, \mu_1^R(D) = b_2, \end{aligned}$$

where $a_1 + b_1 \leq 1$ and $a_2 + b_2 \leq 1$. Calculating the CEU from every action leads to the following conditions:

$$\begin{aligned} \text{type } t_1 &: \quad a_2 = \frac{1 + 3b_1}{2} \\ \text{type } t_2 &: \quad a_1 = \frac{1}{2} \end{aligned}$$

where $b_1 \leq 1/3$. Although we may choose any parameters satisfying these conditions, consider the following case

$$\begin{aligned} \mu_1^L(U) &= 1/2, \mu_1^L(D) = 1/6, \\ \mu_1^R(U) &= 4/3, \mu_1^R(D) = 1/6. \end{aligned}$$

Now suppose that, in the equilibrium, type t_1 is mixing with a probability of q_1 and type t_2

with a probability of q_2. Player 2's indifferent conditions are calculated by updated capacities:

$$L \text{ is observed :} \qquad \frac{4\beta_1}{\beta_1+1-\alpha_3}=\frac{2\beta_1+1-\alpha_3}{\beta_1+1-\alpha_3}$$

$$R \text{ is observed :} \qquad \frac{\alpha_2}{\alpha_2+1-\beta_4}=\frac{\beta_2}{\beta_2+1-\alpha_4}$$

These are reduced to

$$\beta_1=\frac{1-\alpha_3}{2} \tag{3}$$

$$\alpha_2(1-\alpha_4)=\beta_2(1-\beta_4), \tag{4}$$

which are also compatible with convexity requirements.

To illustrate this equilibrium, set $\beta_1=1/8$ and $\alpha_3=3/4$ satisfying (3), and $\alpha_2=1/2$, $\alpha_4=0$, $\beta_2=1/3$, and $\beta_4=1/4$ satisfying (4). The consistency of belief sets implies that the probability distribution generated by (q_1, q_2) and $\pi=(1/2, 1/2)$ have to be contained in $\mathcal{P}_2$, we have

$$0\le q_1\le 1/4 \quad\text{and}\quad 1/2\le q_2\le 2/3, \quad\text{or}$$
$$3/4\le 1-q_1\le 1 \quad\text{and}\quad 1/3\le 1-q_2\le 1/2$$

On the other hand, the updated belief set for player 2 is now:

$$\begin{aligned}
\underline{\mu}_2^L(t_1) &= 0, \overline{\mu}_2^L(t_1)=3/7\\
\underline{\mu}_2^L(t_2) &= 4/7, \overline{\mu}_2^L(t_2)=1\\
\underline{\mu}_2^R(t_1) &= 2/5, \overline{\mu}_2^R(t_1)=3/4\\
\underline{\mu}_2^R(t_2) &= 1/4, \overline{\mu}_2^R(t_2)=3/5.
\end{aligned}$$

When p's marginal distribution is calculated as:

$$\begin{aligned}
0 &\le \frac{q_1}{q_1+q_2}\le\frac{3}{11}\\
\frac{2}{3} &\le \frac{q_2}{q_1+q_2}\le 1\\
\frac{3}{5} &\le \frac{1-q_1}{(1-q_1)+(1-q_2)}\le\frac{3}{4}\\
\frac{1}{4} &\le \frac{1-q_2}{(1-q_1)+(1-q_2)}\le\frac{2}{5}
\end{aligned}$$

It is verified that these range is also contained in updated belief sets.

The existence of a completely mixed equilibrium suggests that there is also a set of hybrid equilibria even if player 1 is the expected utility maximizer.

Hybrid Equilibrium Consider the following strategy of player 1, type t_1 is mixing L and R, type t_2 choose the pure action L.

Then, $\beta_2=0$ and $\beta_3=0$.

Suppose that player 2 observed R. Then, 2's updated belief set is represented as:

$$\begin{aligned} \underline{\mu}_2^R(t_1) &= \frac{\alpha_2}{\alpha_2+1-\beta_4}, \quad \underline{\mu}_2^R(t_2)=0. \\ \overline{\mu}_2^R(t_1) &= 1, \quad \overline{\mu}_2^R(t_2)=\frac{1-\beta_4}{\alpha_2+1-\beta_4}. \end{aligned}$$

Given these beliefs, so that player 2 chooses mix strategy, the CEU from U and D have to be equal:

$$\frac{\alpha_2}{\alpha_2+1-\beta_4}=0 \Rightarrow \alpha_2=0.$$

Suppose player 2 chooses U with probability q. Then, type t_1 is indifferent between L and R if $q=\frac{1}{2}$.

On the other hand, suppose that player 2 observed L. Then, player 2's updated belief set is given as:

$$\begin{aligned} \underline{\mu}_2^L(t_1) &= \frac{\alpha_1}{\alpha_1+1}, \underline{\mu}_2^L(t_2)=\frac{\beta_1}{\beta_1+1-\alpha_3}. \\ \overline{\mu}_2^L(t_1) &= \frac{1-\alpha_3}{\beta_1+1-\alpha_3}, \overline{\mu}_2^L(t_2)=\frac{1}{\alpha_1+1}. \end{aligned}$$

The consistency requirement for belief sets gives $\overline{\mu}_2^L(t_2)=1$, i.e. $\alpha_1=0$. Given this beliefs, the CEU is:

$$\begin{aligned} &\text{play } U: \quad \frac{4\beta_1}{\beta_1+1-\alpha_3} \\ &\text{play } D: \quad 1 \end{aligned}$$

Therefore, when $\beta_1 \geq \frac{1-\alpha_3}{3}$, player 2 chooses to play U, which is conformable to player 1's choice.

Therefore, to summing up,

$$\alpha_1=\alpha_2=\alpha_4=0, \beta_2=\beta_3=0, \beta_4 \geq \beta_1, \text{and } \beta_1 \geq \frac{1-\alpha_3}{3},$$

which supports the hybrid EMP.

5 Concluding Remarks

Equilibria with multiple priors examined in this paper are strongly influenced by the choice of an update rule and the representation of initial priors, i.e. forms of convex capacities. To minimize the lack of robustness, adopting the general form of capacities would be recommended, especially in making use of the Dempster-Shafer rule.

It is also shown that, in general, some equilibrium behavior other than Nash equilibria appears, such as hybrid or complete mixed equilibria. The notion of EMP might gives some foundation for such behavior actually observed in real economic situations, and enriches the patterns of equilibrium behavior to explain them in proper applications. However, in view of theoretical justifications, the EMP might considerably broaden the set of equilibria beyond Nash equilibria. In this sense, some kind of the selection mechanism for EMP would be needed for reinforcement.

References

Crawford, V. "Equilibrium without Independence,“*Journal of Economic Theory*, vol. 50, Issue 1, February, 1990, pp. 127-154.

Dempster, A. P. “Upper and Lower Probabilities Induced by a Multivalued Mapping”, *Annals of Mathematical Statistics*, 38, 1967, pp. 205-47.

Denneberg, D. “Conditioning (Updating) Non-Additive Measures,” *Annals of Operations Research*, vol. 52, 1994, pp. 21-42.

Dow, J. and S. R. C. Werlang. “Nash Equilibrium under Knightian Uncertainty: Breaking Down Backward Induction,” *Journal of Economic Theory*, vol. 64, 1994, pp. 305-324.

Eichberger, J. and D. Kelsey. “Sequential Two-Player Games with Ambiguity,” *International Economic Review*, vol. 45., Issue 4, November, 2004, pp. 1229-1261.

Eichberger, J. and D. Kelsey. “Non-Additive Beliefs and Strategic Equilibria,” *Games and Economic Behavior*, vol. 30, 2000, pp. 183-215.

Fagin, R. and J. Y. Halpern, “A New Approach to Updating Beliefs”, in *Uncertainty in Artificial Intelligence 6*, ed. by P.P. Bonissone, M. Henrion, L.N. Kanal, and J.F. Lemmer, 1991, pp. 347-374.

Gilboa, I. and D. Schmeidler, “Updating Ambiguous Beliefs,” *Journal of Economic Theory*, vol. 59, Issue 1, February, 1993, pp. 33-49.

Kajii, A. and T. Ui. “Incomplete Information Games with Multiple Priors,” *The Japanese Economic Review*, vol. 56, No. 3, September, 2005, pp. 332-351.

Lo, K. C. “Extensive Form Games with Uncertainty Averse Players,” *Games and Economic Behavior*, vol. 28, 1999, pp. 256-270.

Marinacci, M. “Ambiguous Games,” *Games and Economic Behavior*, 31, 2000, pp.191-219.

Ryan, M. J. “Violations of Belief Persistence in Dempster-Shafer Equilibrium,” *Games and Economic Behavior*, vol. 39, No., , 2002, pp. 167-174.

Shafer, G. *A Mathematical Theory of Evidence*, Princeton University Press, Princeton, 1976.

Walley, P. *Statistical Reasoning with Imprecise Probabilities*, Chapman and Hall, London, 1991.

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